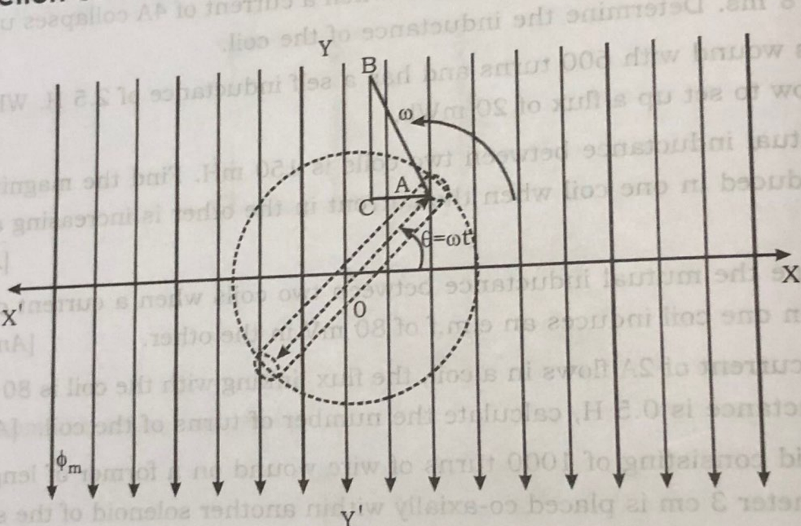
CHAPTER 4

FUNDAMENTALS OF ALTERNATING CURRENT

4.1 Production of Alternating Emf



Consider a single turn rectangular coil rotating with a constant angul velocity of ω radian/second in a uniform magnetic field. The axis of rotation being perpendicular to the magnetic lines of force. Let the time be measur from the instant the coil lies in the plane of reference XOX'. The angle θ swe by the coil in a time t seconds is given by $\theta = \omega t$. Where ω is the angular veloc of the coil in rad/second. Linear velocity v of the coil sides,

$v = \omega r$ m/s where 'r' is the radius of the path in meters.

Linear velocity of coil side at right angles to the magnetic field = AD = v sin $v \sin \omega t$.

When the coil continues its motion in the direction AC,

The flux cut per second = $Blv \sin \omega t$

where

'l' is the length of the coil side parallel to the axis in meters

'B' is the flux density in tesla (weber/m^2)

Thus the emf generated in the coil side at time t = $Blv \sin\omega t$

Emf generated in the coil at time $t = 2Blv \sin\omega t = 2Bl\omega r \sin\omega t$ $(v = \omega r)$

But maximum flux linking the coil, $\phi = B \times 2lr$.

Thus emf generated in the coil at any instant 't' $= \phi\omega \sin\omega t$ $(\phi = 2Blr)$ ....(1)

Thus emf will be maximum when $\sin\omega t = 1$

Hence maximum value of emf generated, $E_m = \phi\omega$

Substituting it in equation (1)

The instantaneous value of emf generated at any time t is

$$e = E_m \sin\omega t \text{ ....(2)}$$

Similarly the expression for induced alternating current is given by,

$$i = I_m \sin\omega t \text{ ....(3)}$$

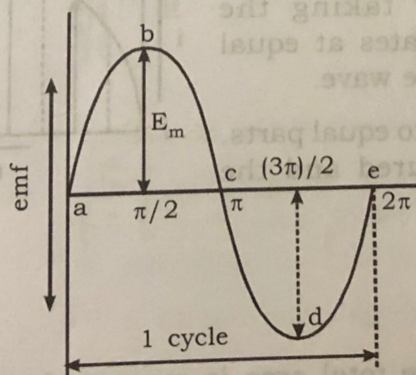
If 'f' is the frequency of rotation of the coil. i.e., no.of cycles passed through per second, then $\omega = 2\pi f$

Substituting for ω in equation (2) & (3)

$$e = E_m \sin(2\pi f)t$$

$$i = I_m \sin(2\pi f)t$$

If the emf values as given by equation (2) from instant to instant are plotted along the Y-axis and time along x-axis, the graph will be as shown in the figure.



Sine Wave of emf

The graph of voltage shown in the figure is called a sinusoidal alternating emf. The trace abcde of the graph completes one cycle and consists of two alternatives, one positive and other negative. Such a wave will complete a certain number of cycles in one second, which is called the frequency of the wave and is expressed in Hertz(Hz). In practice while drawing the ac waves, horizontal axis is marked in radians or degrees instead of seconds.

4.2 Important Terms

Cycle :- One complete set of positive and negative values of an alternating quantity is called a cycle.

Periodic time: The time taken for one cycle is known as time period or periodic time (T). The relationship between frequency and time period is given by,

$$T = \frac{1}{f}$$

Frequency : The number of cycles completed in one second is called the frequency of an alternating quantity. Frequency is expressed in cycles/second or Hertz.

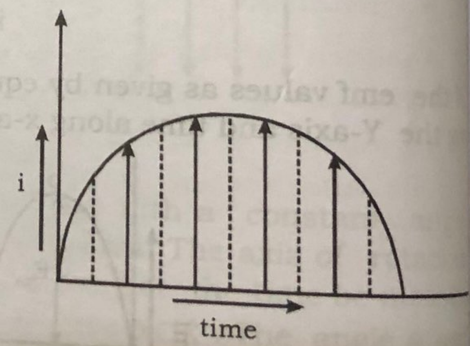
Amplitude : This is the magnitude of the maximum positive or negative value of alternating quantity. It is often referred to as the peak value.

Instantaneous value : The value of voltage or current at a particular instant is known as instantaneous values.

Average value : The average value of the current or voltage of an alternating wave shape is the arithmetic mean of the ordinates at equal intervals over a half cycle of the wave. However, if the arithmetic mean is found out over the complete cycle, it will be zero for sinusoidal as well as for non-sinusoidal wave, provided the wave shape is symmetrical.

4.3 Determination of Average Value

i) Mid Ordinate Method :- The average value of an alternating wave can be determined graphically by taking the arithmetic mean of the ordinates at equal intervals over a half cycle of the wave.



Let the wave from be split into equal parts. The middle values are measured and the average value is calculated.

$$I_{av} = \frac{i_1 + i_2 + i_3 + \ldots.. + i_n}{n}$$

ii) Analytical method :- The total area is measured over the half cycle is divided by the base gives the average value.

Let us assume a sine wave of current $i = I_m \sin\theta$

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The area of the sine curve from 0^0 to 180^0 may be formed out by integrating the curve.

$$\text{Area} = \int_0^{180} i\,dt = \int_0^{180} I_m \sin\theta\, d\theta$$

$$= I_m\,[-\cos\theta]_{0^0}^{180^0} = 2I_m$$

$$\text{Base of the wave from} = \pi \text{ radian}$$

$$\therefore I_{av} = \frac{\text{Area}}{\pi} = \frac{2I_m}{\pi}$$

$$= \frac{2}{\pi} \times I_m = 0.637\, I_m$$

4.4 Root Mean Square Value (RMS Value)

RMS value of an alternating current may be defined as that value of dc current which when flows through a given resistance produces the same amount of heat as that produced by the alternating current passing through the same resistance for the same time. RMS value is also called the effective or virtual value.

Heat produced by ac current and dc current are compared, the resistance and time being kept constant. The value of d.c current which produces the same heat as the unknown ac current is known as r.m.s value of a.c.

Heat is proportional to I^2. Therefore, the a.c value is squared first, then average or mean value is found out. Finally the root is taken to give the effective value.

Let us take the simple wave form $i = I_m \sin\theta$

$$I_{r.m.s.} = \sqrt{\int_0^{\pi} i^2\, dt} = \sqrt{\frac{\int_0^{\pi} (I_m \sin\theta)^2\, d\theta}{\pi}}$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta} = \sqrt{\frac{I_m^2}{\pi} \times \frac{\pi}{2}}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$\text{R.M.S. Value} = \frac{\text{Maximum value}}{\sqrt{2}}$$

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Form factor :- Form factor of an alternating wave is defined as the ratio of its rms value to the average value.

$$\text{Form factor} = \frac{\text{Rms Value}}{\text{Average Value}} = \frac{0.707 \times \text{Maximum Value}}{0.637 \times \text{Maximum Value}}$$

$$= \frac{0.707}{0.637} = 1.11 \text{ for sine wave}$$

Peak factor :- Peak factor of an alternating wave is defined as the ratio of its maximum value to the rms value.

$$\text{Peak factor} = \frac{\text{Maximum Value}}{\text{rms Value}} = \frac{\text{Maximum Value}}{0.707 \times \text{Maximum Value}}$$

$$= 1.414.$$

4.5 Phasor representation

Any alternating quantity can be represented by a rotating Phasor. When several alternating currents or voltages are evolved, there would be definite phase relationships between them. Phasors can be expressed mathematically in the following forms.

1. Rectangular form
2. Trigonometric form
3. Exponential form
4. Polar form

The j Operator

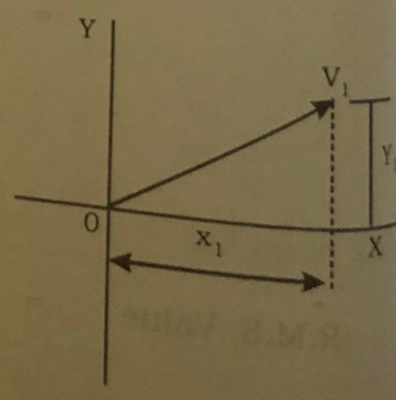
The letter 'j' is used to express operation of counter clock-wise rotation of a vector through 90^0. If this operation is done twice on a vector, the vector gets rotated counter clock wise through 180^0 and reverse its sine. I.e., gets multiplied by -1.

Thus $j \times j = j^2 = -1$. Hence $j = \sqrt{-1}$

Thus $j^2V = -V$.

Rectangular form:-

Any vector may be resolved into X-component and Y-component. The vector V_1 is x_1 and y-component is y_1 i.e., $\vec{V_1} = x_1 + jy_1$. In language of mathematics, x_1 is the real component and y_1 is the imaginary component. Numerical value of vector V_1 is $\sqrt{x_1^2 + y_1^2}$ While its angle with the x-axis is given by $\tan^{-1}(y_1/x_1)$.



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Trigonometric form :-

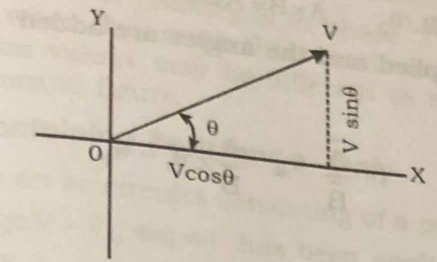


Figure shows the vector v and its x-component is v cos θ and its y-component is vsin θ. Hence we may write

$$\vec{v} = v(\cos\theta + j\sin\theta)$$

Exponential form :-

Euler's equation states that

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

i.e., $e^{+j\theta} = \cos\theta + j\sin\theta$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Hence vector v may be expressed in the exponential form.

$$\vec{v} = ve^{+j\theta}$$

Polar form :-

Consider the vector v written in trigonometric form $v(\cos\theta + j\sin\theta)$. This vector makes an angle θ with the positive x-axis and has magnitude v. Hence it may be written as $v\angle\theta$ where, angle θ is taken counter clock wise. Similarly vector $v(\cos\theta - j\sin\theta)$ may be expressed as $v\angle-\theta$.

Addition and Subtraction

The rectangular form of representing phasors is best suited for addition and subtraction operations. Consider two phasors $\vec{A} = a_1 + ja_2$ $\vec{B} = b_1 + jb_2$

$$\vec{A} + \vec{B} = (a_1 + b_1) + j(a_2 + b_2)$$

$$\vec{A} - \vec{B} = (a_1 - b_1) + j(a_2 - b_2)$$

Multiplication and Division

The polar form of representing phasors is best suited for multiplication and division. Consider two phasors.

$\vec{A} = A\angle\theta_1$ and $\vec{B} = B\angle\theta_2$ $\quad \vec{A}\times\vec{B} = AB\angle\theta_1 + \theta_2$

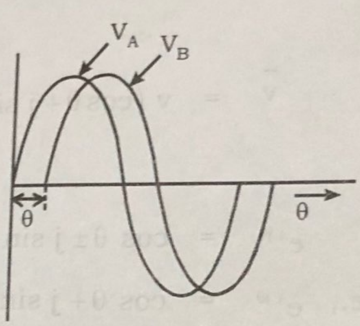
The magnitudes are multiplied and the angles are added

Division :-

$$\frac{\vec{A}}{\vec{B}} = \frac{A}{B}\angle\theta_1 - \theta_2$$

4.6 Phase Difference

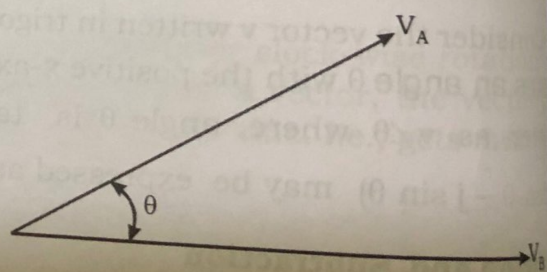
Voltage A is represented by a sine wave and the other voltage B is also a sine wave.



But voltage B starts after an interval of θ. If we represents

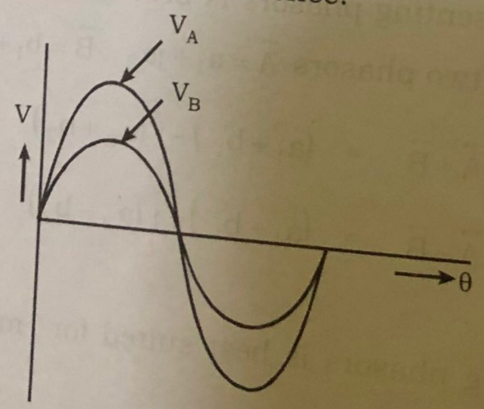
$$V_A = V_m \sin\omega t$$
$$V_B = V_m (\sin \omega t - \theta)$$

Minus sign indicates that voltage B starts after θ^0. It is said to be lagging.



The voltage A is leading the voltage B by an angle θ or we can say that the voltage B is lagging the voltage A by an angle θ.

The difference in angle between two voltage is known as phase difference.



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The other terms used in phasors is in phase. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum values may be different in magnitude. Two such voltage wave forms are shown in figure.

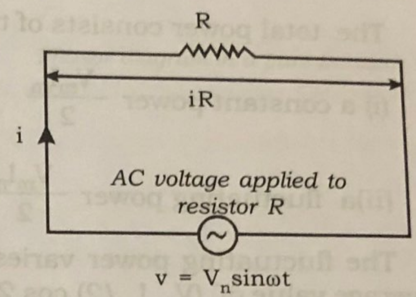
4.7 A.C Circuit Containing Resistance Only

The figure shows an ac circuits consisting of a pure resistance (RΩ) , to which an alternating voltage $V = V_m \sin \omega t$ has been applied. The instantaneous value of current is given by,

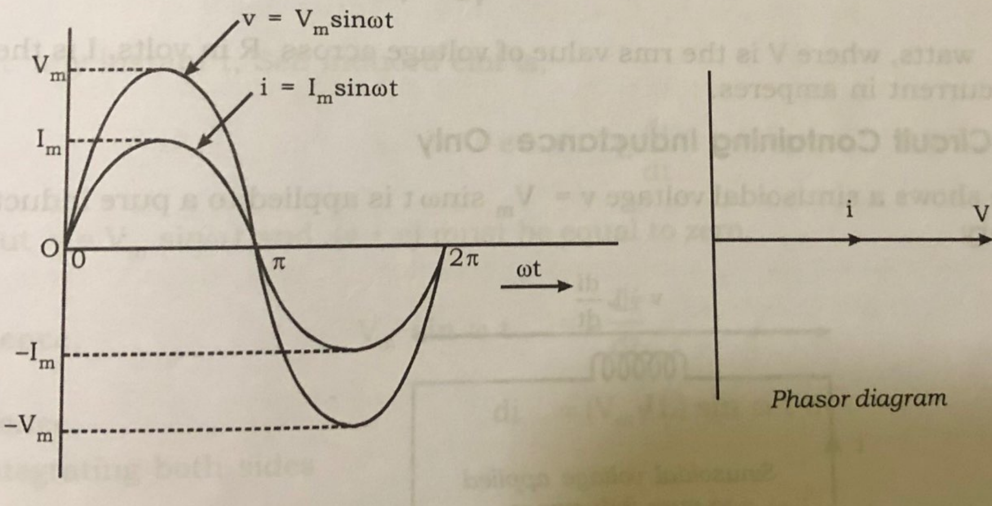
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$= \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$\left(\text{Where } I_m = \frac{V_m}{R}\right)$$



Here, the voltage and current are in phase.



Power in resistive circuits

Power is drawn by the circuit at any instant is the product of the instantaneous voltage and instantaneous current.

$$P = v \times i = V_m \sin \omega t \times I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

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$$= V_m I_m \frac{(1 - \cos 2\omega t)}{2}$$

$$= \frac{1}{2} V_m I_m (1 - \cos 2\omega t)$$

$$= \frac{1}{2} V_m I_m - \frac{1}{2} V_m I_m \cos 2\omega t$$

The total power consists of two parts.

(i) a constant power $\frac{V_m I_m}{2}$

(ii)a fluctuating power $\frac{V_m I_m}{2} \cos 2\omega t$

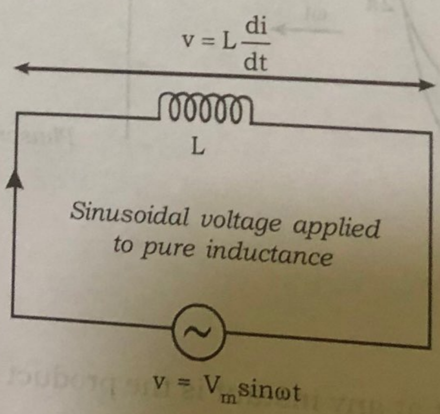
The fluctuating power varies at a frequency 2ω. Over a complete cycle, t average value of $(V_m I_m/2) \cos 2\omega t$ is zero. Hence over a complete cycle, the pow is given by,

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

P = VI watts, where V is the rms value of voltage across R in volts. I is the r value of current in amperes.

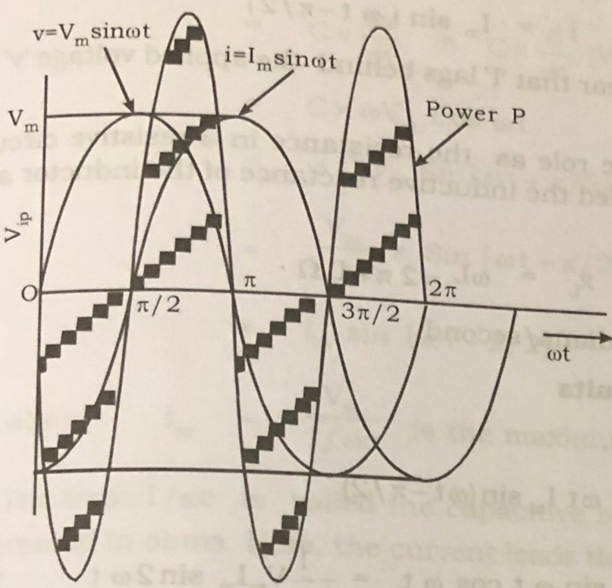
4.8 A.C Circuit Containing Inductance Only

Figure shows a sinusoidal voltage $v = V_m \sin\omega t$ is applied to a pure inductan of L Henry.

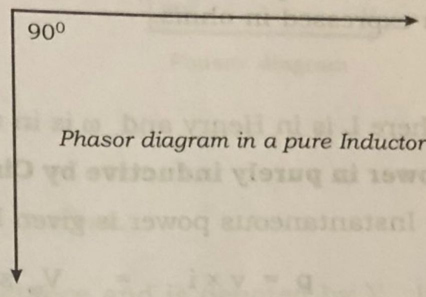


Sinusoidal voltage applied to pure inductance

Then a back emf is produced due to the self inductance of the inductor, Th back emf at any time 't' opposes the change of current through the inductor. T inductor does not contain any resistive element. Hence the entire applied volta has to overcome the self induced emf alone.



Variation of Voltage, Current and Power with ωt in a pure Inductor



Phasor diagram in a pure Inductor

At any instant t, Self induced emf is,

$$e = -L\frac{di}{dt}$$

But $v = V_m \sin\omega t$ and $(v + e)$ must be equal to zero.

Hence, $V_m \sin \omega t = L\frac{di}{dt}$

Hence, $di = (V_m / L) \sin \omega t \, dt$

Integrating both sides

$$i = \frac{V_m}{L}\int \sin \omega t \, dt = \frac{V_m}{L} \times \frac{-\cos \omega t}{\omega}$$

$$= -\frac{V_m}{\omega L}\cos \omega t = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

when $\sin(\omega t - \pi/2)$ is unity, current is maximum and is denoted by I_m. Then,

$$I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$$

Hence,

$$i = I_m \sin(\omega t - \pi/2)$$

From the above equation it is clear that 'I' lags behind the applied voltage 'v' by 90^0.

The quantity ωL plays the same role as the resistance in a resistive circuit. Hence this quantity ωL or X_L is called the inductive reactance of the inductor and is expressed in ohms.

$$X_L = \omega L = 2\pi f L \,\Omega.$$

where L is in Henry and ω is in radians/second.

Power in purely inductive by Circuits

Instantaneous power is given by,

$$p = v \times i = V_m \sin \omega t \, I_m \sin(\omega t - \pi/2)$$

$$= -V_m I_m \sin \omega t \cos \omega t = -\frac{1}{2} V_m I_m \sin 2\omega t$$

Average power for one complete cycle

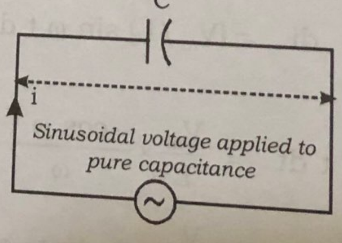
$$p = \frac{-V_m I_m}{2} \times \text{average of } (\sin 2\omega t) = 0$$

Hence the total power consumed by a purely inductive circuit is zero.

4.9 A.C. Circuit Containing Capacitance Only

$$v = V_m \sin \omega t$$

Figure shows an a.c circuit containing a capacitor of capacitance 'C' farads. Let an alternating voltage given by,



$v = V_m \sin\omega t$

$v = V_m \sin \omega t$ be applied to the above circuit. Charging current in the capacitor circuit is given by,

$$i = C \times \text{Rate of change of potential difference}$$

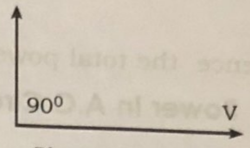
$$= C \times \frac{dv}{dt} = C \times \frac{d}{dt}(V_m \sin \omega t)$$

$$= C \times \omega V_m \cos \omega t$$

$$= \omega C V_m \sin(\omega t + \pi/2)$$

$$= \frac{V_m}{1/\omega c} \times \sin(\omega t + \pi/2)$$

$$= I_m \sin(\omega t + \pi/2)$$



Phasor diagram

where $I_m = \frac{V_m}{1/\omega c}$ is the maximum current.

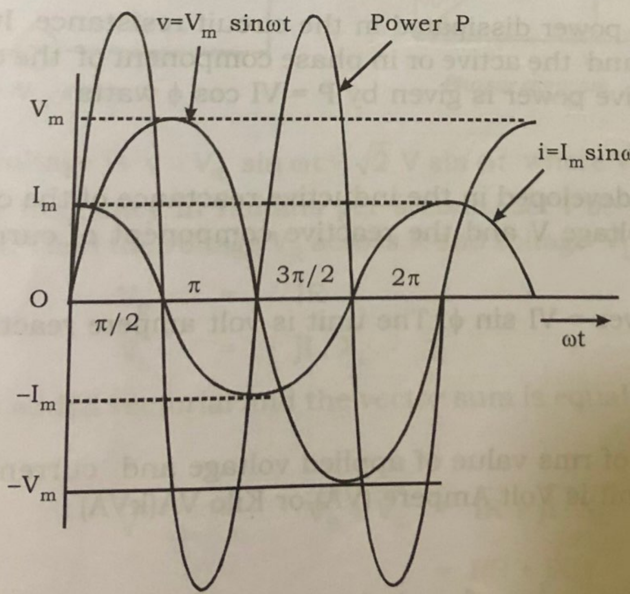
The term $1/\omega c$ is called the capacitive reactance and is denoted by X_c. It is expressed in ohms. Here, the current leads the applied voltage by an angle $\pi/2$ or 90^0.

Power in a purely capacitive circuit

Instantaneous power is given by

$$P = v \times i = V_m \sin \omega t \times I_m \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin \omega t \cos \omega t = \frac{1}{2} V_m I_m \sin 2\omega t$$



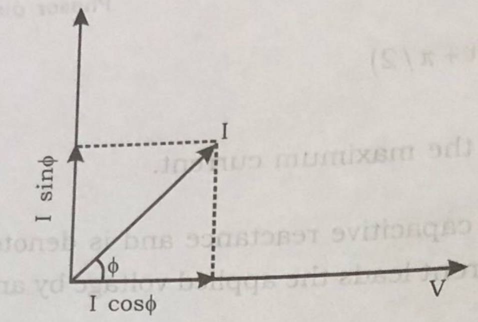
Variation of voltage, current and power in a pure capacitor

Average power for one cycle is given by

$$P = \frac{1}{2} V_m I_m \times \text{average of } (\sin 2\omega t) = \text{zero.}$$

Hence the total power consumed by a purely capacitive circuit is zero.

4.10 Power In A.C Circuit



Current I leads the voltage by an angle ϕ. We may resolve 'I' into two mutuall perpendicular components, namely I cos ϕ along V and I sin ϕ perpendicular to as shown in figure. The component I cos ϕ is in phase with the applied voltage an is therefore called the in-phase component or active component. The componen I sin ϕ is quadrature with the applied voltage and is therefore called the quadratur component or reactive component.

Active Power or real power

This is the actual power dissipated in the circuit resistance. It is given by th product of voltage V and the active or in phase component of the current throug the circuit. Thus active power is given by P = VI cos ϕ watts

Reactive power

This is the power developed in the inductive reactance of the circuit. It is give by the product of voltage V and the reactive component of current through th circuit.

Thus reactive power = VI sin ϕ. The unit is volt ampere reactive. (VAR) or kil VAR(kVAR).

Apparent power

It is the product of rms value of applied voltage and current. Thus apparen power = V × I. The unit is Volt Ampere (VA) or Kilo VA(kVA)

Power factor

It is defined as the cosine of angle between voltage and current in a circuit.

Power factor = cos ϕ

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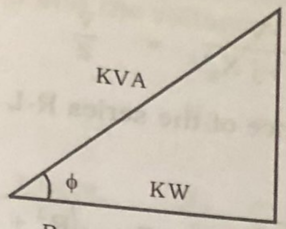


Power triangle

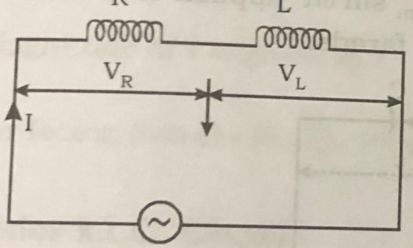
It is also defined as the ratio of real power to apparent power

$$\text{Power factor} = \frac{\text{Real power}}{\text{Apparent power}}$$

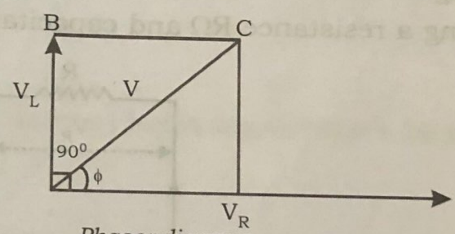
$$= \frac{VI\cos\phi}{VI} = \cos\phi$$

4.11 A.C Current Through Resistance And Inductance (R-L Circuit)

The figure shows a sinusoidal voltage v applied to a pure inductance L Henry with series resistance R ohm.



$v = V_m \sin\omega t$



Phasor diagram

The applied voltage is $v = V_m \sin\omega t = \sqrt{2}\, V \sin\omega t$ where V is the rms value and ω is the angular frequency in radians per second. Let I be the rms value of the resulting current. Then the voltage V_R across R and voltage V_L across L are given by

$$V_R = IR$$

$$V_L = jI \cdot X_L$$

V_R and V_L are added vectorial and the vector sum is equal to the applied voltage V.

i.e., $$\vec{V} = \overrightarrow{V_R} + \overrightarrow{V_L} = \overrightarrow{IR} + \overrightarrow{jI \cdot X_L}$$

$$= I(R + jX_L)$$

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Hence $\quad I = \dfrac{V}{R + j X_L} = \dfrac{V}{Z}$

where $Z = (R + jX_L)$ is the impedance of the series R-L circuit.

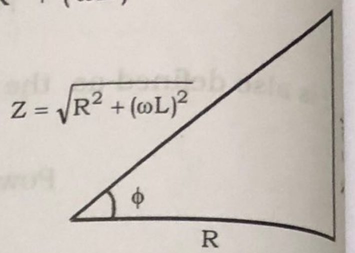
Magnitude of Z is given by,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$$

Hence, magnitude of current is given by

$$I = \frac{V}{Z}$$

Power factor $(\cos\phi) = \dfrac{R}{Z}$

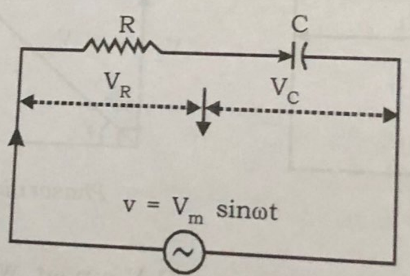


Impedance triangle for series R-L circuit

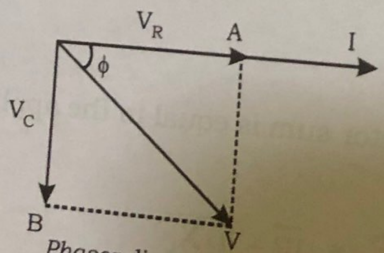
Hence, the voltage V leads the current I by angle ϕ

4.12 A.C Current Through Resistance And Capacitance (R-C Circuit)

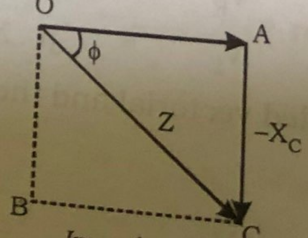
The figure shows a sinusoidal voltage $v = V_m \sin \omega t$ applied to a series circuit containing a resistance RΩ and capacitance C farads.



Series RC circuit



Phasor diagram



Impedance triangle

Applied Voltage $\quad v = V_m \sin \omega t = \sqrt{2} V \sin \omega t$

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where V is the rms value of the applied voltage. Let I be the resulting rms current. The voltage across R(V_R) and the voltage across C(V_c) are given by

$$V_R = IR$$

$$V_C = -jI \cdot X_C$$

where, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

The applied voltage V is the vector sum of V_R and V_C.

i.e., $V = \vec{V_R} + \vec{V_C} = IR - jI.X_C = I(R - jX_C)$

Hence $I = \frac{V}{(R - jX_C)} = \frac{V}{\left(R + \frac{1}{j\omega C}\right)} = \frac{V}{Z}$

where $Z = \left(R + \frac{1}{j\omega C}\right)$ is the impedance of the series R.C circuit.

Magnitude of Z $= \sqrt{R^2 + Xc^2}$

Magnitude of I is given by $I = \frac{V}{Z}$

Power factor $(\cos\phi) = (R/Z)$. Here the current I leads the voltage V by an angle ϕ^0.

4.13 Series R.L.C. Circuit

Fig.1 shows an a.c circuit in which resistance R, inductance L and capacitance C are connected in series. An ac supply at a frequency f hertz is applied to the circuit.

Let v be the rms value of the voltage applied to the circuit,

V_R the rms value of the voltage across the resistance R,

V_L the rms value of the voltage across the inductance L,

V_C the rms value of the voltage across the capacitance C, and

I the rms value of current flowing through the circuit.

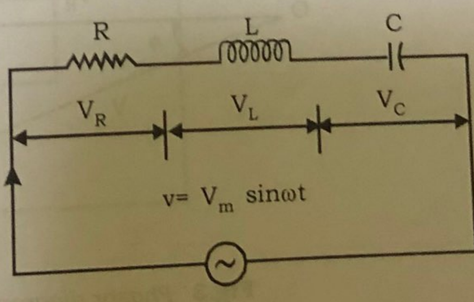

Fig. 1 *RLC circuit*

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The phasor diagram of the circuit is shown in fig(2) taking the rms value of current as a reference vector. It is represented by OA. The voltage V_L leads the current by 90^0 and is shown by phasor BC. The voltage V_C lags the current by 90^0 and is shown by phasor BD. Phasors BC and BD are in direct opposition and hence their resultant is given by

$$BE = BC - BD \text{ (assuming } BC > BD)$$

Hence, the applied voltage will be the phasor sum of OB and BE, namely OE.

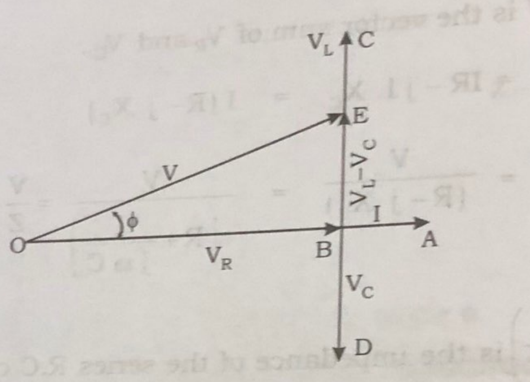

Fig.2 *Phasor diagram of R-L-C circuit $V_L > V_C$*

$$(OE)^2 = (OB)^2 + (BE)^2 = (OB)^2 + (BC - BD)^2$$

$$V^2 = V_R^2 + (V_L - V_C)^2 = (IR)^2 + (IX_L - IX_C)^2 = I^2 [R^2 + (X_L - X_C)^2$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

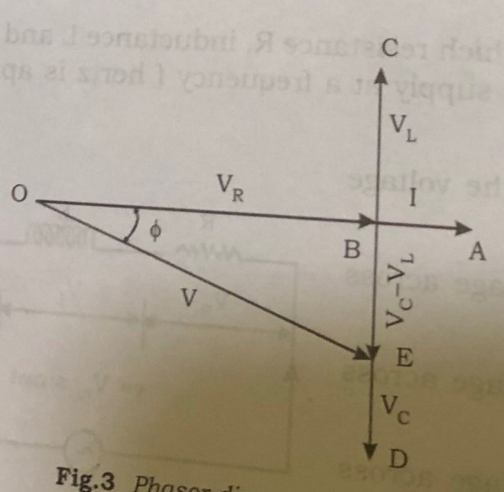

Fig.3 *Phasor diagram of R-L-C circuit $V_L < V_C$*

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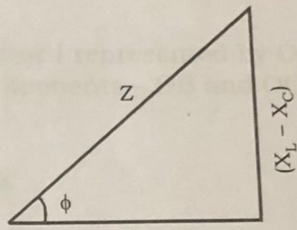
$$\therefore \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

where Z is the impedance of R.L.C circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

As the voltage drop across the inductance has been assumed grater than the voltage across the capacitance, the resultant circuit becomes inductive. Hence the current in the circuit lags the applied voltage V by an angle ϕ. The power factor of such a circuit is then lagging.

Power factor of the circuit, $\cos\phi = \frac{R}{Z}$



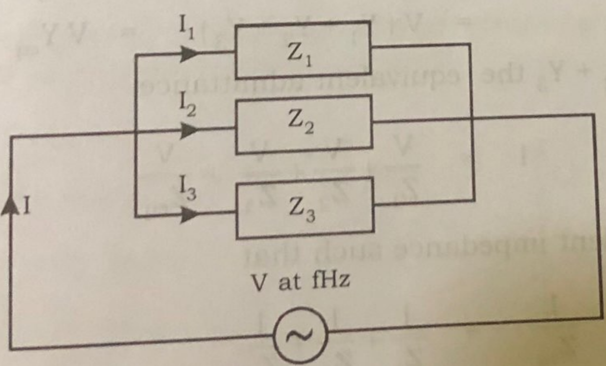
Impedance triangle for series R–L–C circuit

In case the voltage across the capacitance is greater than the inductance, the resultant circuit becomes capacitive. The phasor diagram is shown in figure(3). Thus the power factor of such circuit is leading. Hence if the resultant reactance is positive, the current lags behind the applied voltage (lagging power factor) and if it is negative the current leads the applied voltage (leading power factor).

4.14 Parallel Circuits

Consider a parallel circuit consisting of 3 impedances z_1, z_2, z_3. Let an alternating voltage V at frequency f Hz be applied to the circuit. Let I be the total current, and I_1, I_2, I_3, be the branch current as shown in figure.

$$I = I_1 + I_2 + I_3 \text{ the phasor sum}$$



But $\quad I_1 = \frac{V}{Z_1}$

$$I_2 = \frac{V}{Z_2}$$

$$I_3 = \frac{V}{Z_3}$$

Admittance :- Admittance is defined as the reciprocal of impedance i.e., $Y = \frac{1}{Z}$

Its unit is mho.

Let Y_1, Y_2 and Y_3 denote the branch admittance we have,

$$Y_1 = \frac{1}{Z_1}$$

$$Y_2 = \frac{1}{Z_2}$$

$$Y_3 = \frac{1}{Z_3}$$

$$I_1 = V\left[\frac{1}{Z_1}\right] = V \cdot Y_1$$

$$I_2 = V\left[\frac{1}{Z_2}\right] = V \cdot Y_2$$

$$I_3 = V\left[\frac{1}{Z_3}\right] = V \cdot Y_3$$

Total current $I = I_1 + I_2 + I_3 = VY_1 + VY_2 + VY_3$

$$= V(Y_1 + Y_2 + Y_3) = V Y_{eq}$$

where $Y_{eq} = Y_1 + Y_2 + Y_3$ the equivalent admittance

$$I = \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3} = \frac{V}{Z_{eq}}$$

where Z_{eq} = Equivalent impedance such that

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$I = V\left[\frac{1}{Z_{eq}}\right] = V Y_{eq}$$

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Since $1/Z_{eq} = Y_{eq}$

Thus, if the branch admittance and the total admittance are known, then we have

Branch Current $I_1 = V Y_1$

Branch Current $I_2 = V Y_2$

Branch Current $I_3 = V Y_3$

Branch Current $I = V Y_{eq}$

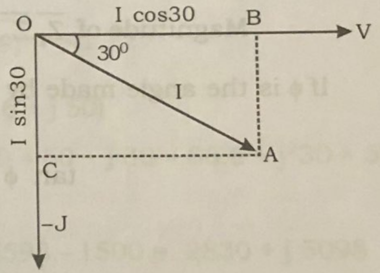
PROBLEMS

1. A current vector of magnitude 100 A is (a) lagging the voltage vector by 30^0 (b) leading the voltage vector by 30^0 and (c) is in phase with the voltage vector. Represent the current in j form.

Solution. (a) Let OA = 100A [Refer Fig]. If current vector I represented by OA lags the voltage vector by an angle 30^0, it has two components – OB and OC.

OB is in phase with V and is equal to OA cos 30.

OC is in quadrature with V and is equal to OA sin 30.

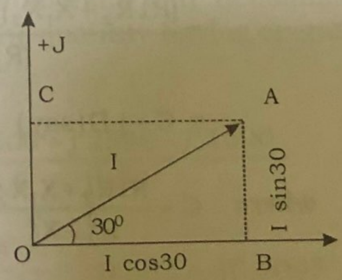


$$\therefore \overline{OB} = OA \cos 30 = I \cos 30 = 100 \cos 30.$$

$$\overline{OC} = -j\, OA \sin 30 = -j \times 100 \sin 30$$

$$\bar{I} = \overline{OB} + \overline{OC} = 100 \cos 30 - j \times 100 \sin 30 = 100\,[\cos 30 - j \sin 30].$$

(b) When the current vector OA leads the voltage vector by ϕ^0, it has 2 components OB, in phase with V and OC in quadrature with V.



$$\overline{OB} = OA \cos\phi = I \cos\phi = 100 \cos\phi$$

$$OC = +J\, OA \sin\phi = +j \times 100 \sin\phi$$

$$I = \overline{OA} = \overline{OB} = \overline{OC} = 100 \cos\phi + j \times 100 \sin\phi$$

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$$\bar{I} = 100\,[\cos\phi + j\sin\phi]$$

But $\phi = 30^0$

$$\bar{I} = 100\,[\cos 30 + j\sin 30]$$

(c) When current vector is in phase with voltage vector there is no compon[ent] in quadrature with voltage vector.

$$\bar{I} = 100 + jO.$$

2. If $\overline{Z_1} = R_1 + jX_1$ and $\overline{Z_2} = R_2 + jX_2$ Find (a) $Z_1 : Z_2$ and (b) $\overline{Z_1}/\overline{Z_2}$

Solution.

$\overline{Z_1} = R_1 + jX_1,\ \overline{Z_2} = R_2 + jX_2$

$$Z_1 Z_2 = (R_1 + jX_1)(R_2 + jX_2) = R_1 R_2 + R_1(jX_2) + (jX_1)R_2 + J^2 X_1 X_2$$

$$= R_1 R_2 + J^2 X_1 X_2 + j\,[R_1X_2 + X_1 R_2] = [R_1 R_2 - X_1 X_2] + j\,[R_1X_2 + X_1 R_2]$$

$$\text{Magnitude of } \overline{Z_1}.\overline{Z_2} = \sqrt{(R_1 R_2 - X_1X_2)^2 + (R_1X_2 + X_1R_2)^2}$$

If ϕ is the angle made by this vector with real axis

$$\tan\phi = \frac{R_1X_2 + X_1R_2}{R_1 R_2 - X_1X_2}$$

or

$$\phi = \tan^{-1}\frac{R_1X_2 + X_1R_2}{R_1 R_2 - X_1X_2}$$

(b) $$\frac{\overline{Z_1}}{\overline{Z_2}} = \frac{R_1 + jX_1}{R_2 + jX_2} = \frac{(R_1 + jX_1)(R_2 - jX_2)}{(R_2 + jX_2)(R_2 - jX_2)} = \frac{R1R_2 - J^2X_1X_2 + jX_1R_2 - jX_{[...]}}{(R_2^2 - j^2X_2^2)}$$

$$= \frac{(R1R_2 + X_1X_2) + j\,(X_1R_2 - jR_1X_2)}{(R_2^2 + X_2^2)} = \frac{R1R_2 + X_1X_2}{R_2^2 + X_2^2} + j\frac{X_1R_2 - jR_{[...]}}{R_2^2 + X_{[...]}^2}$$

$$= C + j\,D(\text{say})$$

where $C = \frac{R1R_2 + X_1X_2}{R_2^2 + X_2^2}$ & $D = \frac{X_1R_2 - JR_1X_2}{R_2^2 + X_2^2}$

$$\text{Magnitude of } \frac{\overline{Z_1}}{\overline{Z_2}} = \sqrt{C^2 + D^2}$$

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If ϕ is the angle made by this vector with the real axis,

$$\phi = \tan^{-1}\frac{D}{C}$$

or

$$\phi = \cos^{-1}\frac{C}{\sqrt{C^2 + D^2}}$$

3. If $\bar{I_1} = [50 + j\,30]$ & $\bar{I_2} = [86.6 + j\,50]$ find the j form and magnitude of (a) $\bar{I_1} + \bar{I_2}$ (b) $\bar{I_1} - \bar{I_2}$ (c) $\bar{I_1}.\bar{I_2}$ (d) $\frac{\bar{I_1}}{\bar{I_2}}$

Solution.

(a) $\bar{I_1} + \bar{I_2} = 50 + j\,30 + 86.6 + j\,50 = 136.6 + j\,80$

Magnitude of $\bar{I_1} + \bar{I_2} = \sqrt{136.6^2 + 80^2} = 158$

(b) $\bar{I_1} - \bar{I_2} = (50 + j\,30) - (86.6 + j\,50) = -36.6 - j\,20$

Magnitude of $\bar{I_1} - \bar{I_2} = \sqrt{(-36.6)^2 + (-20)^2} = 41.72$

(c) $\bar{I_1}.\bar{I_2} = (50 + j\,30)\ (86.6 + j\,50)$

$= 50 \times 86.6 + j\,50 \times 50 + j\,30 \times 86.6 + j^2 30 \times 50$

But $J^2 = -1$

$\bar{I_1}.\bar{I_2} = 4330 + j\,2500 + j\,2598 - 1500 = 2830 + j\,5098$

Magnitude of $\bar{I_1}.\bar{I_2} = \sqrt{2830^2 + 5098^2} = 5832$

(d) $$\frac{\bar{I_1}}{\bar{I_2}} = \frac{50 + j30}{86.6 + j50}$$

This expression must be rationalized, i.e., the numerator and denominator are multiplied by the denominator with sign of the quantity having operator j changed.

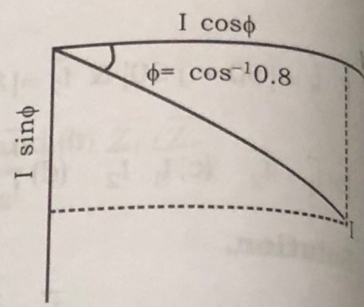
$$\frac{\bar{I_1}}{\bar{I_2}} = \frac{(50 + j\,30)(86.6 - j\,50)}{(86.6 + j\,50)(86.6 - j\,50)} = \frac{4330 - j\,2500 + j\,2598 - j^2\,1500}{86.6^2 - j^2\,50^2}$$

$$= \frac{4330 + 1500 - j\,2500 + j2598}{86.6^2 + 50^2} = \frac{5830 + j98}{10000} = 0.583 + j0.0098^2$$

Magnitude of $\overline{I_1} / \overline{I_2}$ $= \sqrt{0.583^2 + 0.0098^2} = 0.583$

4. When an A.C supply with a supply voltage of 250 V is applied across the circ[uit] the current in the circuit is found to be 25 A. If the current is at 0.8 lagging, find the impedance of the circuit in j form and its magnitude.

Solution : Take the voltage as reference vector.



Then $\overline{V} = 250 + J0$

$\cos\phi = 0.8$

$\phi = \cos^{-1} 0.8 = 36^{0}52'$

$\sin\phi = \sin 36^{0}52' = 0.6$

Sine current vector lags the voltage vector,

$\overline{I} = I[\cos\phi - j\sin\phi] = 25\,[0.8 - j\,0.6] = 20 - j\,15$

Impedance $\overline{Z} = \dfrac{\overline{V}}{\overline{I}} = \dfrac{250 + j\,0}{(20 - j\,15)} = \dfrac{(250 + j\,0)\,(20 + j\,15)}{(20 - j\,15)\,(20 + j\,15)}$

$= \dfrac{5000 + j\,3750}{(20^2 + j\,15^2)} = 8 + j6\Omega$

Numerical value of impedance $= \sqrt{8^2 + 6^2} = 10\,\Omega$

5. An alternating voltage (100 + j60)V at 50 c/s is applied to a circuit havin[g] resistance of 6Ω, inductive reactance of 5 Ω and capacitive reactance of [3 Ω] at 50 c/s. Determine the current in j form and its magnitude.

Solution.

$R = 6\,\Omega$

Inductive reactance $X_L = +j5\,\Omega$

Capacitive reactance $\overline{X_C} = -j\,3\,\Omega$

Total reactance $\overline{X} = \overline{X_L} + \overline{X_C} = +j\,5 - j\,3\,\Omega = +j\,2\,\Omega$

Impedance $\overline{Z} = \overline{R} + \overline{X} = R + JX = (6 + j\,2)\,\Omega$

$V = 100 + j60$

Current, $\overline{I} = \dfrac{\overline{V}}{\overline{Z}} = \dfrac{100 + j60}{6 + j2} = \dfrac{(100 + j\,60)\,(6 - j\,2)}{(6 + j\,2)\,(6 - j\,2)}$

$= \dfrac{600 - j\,200 + j\,360 - j^2\,120}{6^2 + 2^2} = \dfrac{720 + j\,160}{40} = 18 + j4$

Magnitude of current I $= \sqrt{18^2 + 4^2} = 18.44$ A.

6. An alternating voltage [250 + jO] V is applied to a circuit having a resistance of 8 Ω an inductive reactance of 6 Ω at 50 c/s. Find (a) current in j form and its magnitude (b) Power factor and power factor angle of current.

Solution

$$\bar{V} = 250 + j\,0, \quad \bar{X} = +j\,6\,\Omega$$

$$\bar{Z} = \bar{R} + \bar{X} = (8 + j\,6)\,\Omega$$

(a) $$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{250 + j\,0}{8 + j\,6} = \frac{250\,(8 - j\,6)}{(8 + j\,6)\,(8 - j\,6)}$$

$$= \frac{2000 - j\,1500}{8^2 + 6^2} = \frac{2000 - j\,1500}{100} = (20 - j\,15)$$

Magnitude of current $= \sqrt{20^2 + 15^2} = 25$ A.

(b) Power factor of current

$$\cos\phi = \frac{20}{25} \text{ lagging} = 0.8 \text{ lagging}$$

[Current is lagging since the component in quadrature with reference voltage vector is negative]

Power factor angle $= \cos^{-1} 0.8 = 36^0\,52'$

7. An alternating voltage (160 + j 120) V is applied to a circuit and the current in the circuit is found to be (6 + j 8)A. Find (a) the impedance of the circuit (b) the phase angle and (c) the power consumed.

Solution:

$$\bar{V} = (160 + j\,120)\text{ V}, \quad \bar{I} = (6 + j\,8)\text{ A}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{160 + j\,120}{6 + j\,8} = \frac{(160 + j\,120)\,(6 - j\,8)}{(6 + j\,8)\,(6 - j\,8)}$$

$$= \frac{960 - j\,1280 + j720 - j^2\,960}{6^2 + 8^2} = \frac{1920 - j\,560}{100} = 19.2 - j\,5.6$$

(a) Impedance $$Z = \sqrt{19.2^2 + 5.6^2} = 20\,\Omega$$

(b) Phase angle $$= \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{19.2}{20} = 16^0\,16' \text{ leading}$$

[Phase angle is leading since the impedance consists of capacitive reactance –J 5.6 Ω]

$$\text{Voltage} = \sqrt{160^2 + 120^2} = 200 \text{ V}$$

$$\text{Current} = \sqrt{6^2 + 8^2} = 10 \text{ A}$$

$$\text{Power consumed} = VI \cos\phi = 200 \times 10 \cos 16^0\, 16'$$

$$= 2000 \times 0.96 \text{ W} = \mathbf{1920 \text{ watts}}$$

8. If $\bar{I}_1 = 100\angle 30^0$ and $\bar{I}_2 = 60\angle 45^0$, find (a) $\bar{I}_1 . \bar{I}_2$ (b) $\dfrac{\bar{I}_1}{\bar{I}_2}$

Solution:

$$\bar{I}_1 . \bar{I}_2 = 100\angle 30^0 \times 60\angle 45^0 = 6000\angle 75^0$$

$$\frac{\bar{I}_1}{\bar{I}_2} = \frac{100\angle 30^0}{60\angle 45^0} = \frac{100}{60}\angle(30^0 - 45^0) = 1.67\angle -15^0$$

9. Two impedances $Z_1 = (4 + j3)\ \Omega$ and $Z_2 = [6 - j9]\ \Omega$ are connected in series. Find the equivalent impedance in polar form.

Solution:

$$\bar{Z}_1 = (4 + j\,3)\ \Omega,\ \bar{Z}_2 = (6 - j\,9)\ \Omega$$

$$\text{Equivalent impedance} = \bar{Z}_1 + \bar{Z}_2 = 4 + j\,3 + 6 - j\,9 = [10 - j\,6]$$

$$\text{In polar form, } \bar{Z}_1 + \bar{Z}_2 = \sqrt{10^2 + 6^2}\ \angle -\theta$$

$$\text{where } \theta = \tan^{-1}\frac{6}{10} = 30.97^0$$

$$\sqrt{10^2 + 6^2} = 11.66$$

Equivalent impedance in polar form = **11.66 ∠ – 30.97**

10. A sinusoidal alternating current having a frequency of 50 Hz has a peak value of 5A. What is the value of current after 1/300 second from zero?

The instantaneous current equation is given by

$$i = I_m \sin \omega t = I_m \sin 2\pi f t$$

Solution: Here f = 50 Hz, $I_m = 5$ A

i.e.,

$$i = 5 \sin(2\pi \times 50 \times t) = 5 \sin(100\,\pi\,t)$$

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The value of current after 1/300 seconds is

$$i = 5 \sin(100\,\pi\, t) = 5 \sin\left(100 \times 180 \times \frac{1}{300}\right) = 5 \sin 60^0 = \mathbf{4.33\ A}$$

11. An alternating current is given by I = 50 sin (314 t). Find 1) Maximum Value, 2) frequency, 3) time period?

The instantaneous current equation is given by

$$i = (I_m \sin 2\,\pi\, f\, t) \text{.....................(1)}$$

Given equation is,

$$i = 50 \sin(314\, t) \text{.....................(2)}$$

comparing equation (1) and (2)

1. Maximum value, I_m = 50A.
2. Frequency, $f = \dfrac{314}{2\pi} = 50$ Hz.
3. Time period, $T = \dfrac{1}{f} = \dfrac{1}{50} = \mathbf{0.02\ Sec}$

12. A sinusoidal alternating current has a peak value of 20A at 50Hz. How long will it take the current to reach 10A, starting from zero.?

Solution : The current equation is given by

$$i = 20 \sin(2\,\pi\, f\, t)$$

$$10 = 20 \sin(2 \times 180 \times 50 \times t)$$

$$10 = 20 \sin(18000\, t)$$

$$10/20 = \sin(18000\, t)$$

$$1/2 = \text{Sin}(18000\, t)$$

$$18000\, t = \text{Sin}^{-1}(1/2)$$

$$18000\, t = 30^0 = \frac{30}{18000} = \mathbf{1/600\ Sec.}$$

13. For a half wave rectified sinusoidal alternating current, find the following 1) r.m.s value and 2) average value.?

1. RMS Value

$$I = \sqrt{\int_0^{\pi} \frac{i^2\, d\theta}{2\,\pi}} = \sqrt{\int_0^{\pi} \frac{(I_m \sin\theta)^2\, d\theta}{2\,\pi}}$$

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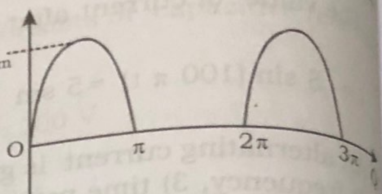


$$= \sqrt{\frac{Im^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta}$$

$$= \frac{Im^2}{4\pi}\int_0^{\pi} (1-\cos 2\theta)\, d\theta = \sqrt{\frac{Im^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{Im^2}{4\pi}\times\pi} \quad = \sqrt{\frac{Im^2}{4}} = \frac{Im}{2}$$

RMS value, I $= \dfrac{\mathbf{I_m}}{\mathbf{2}}$

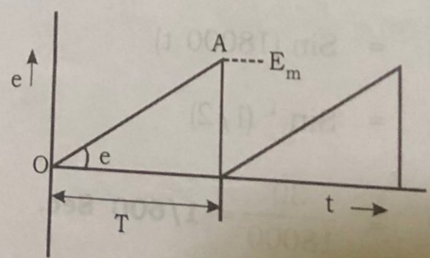
2. Average Value

$$I_{av} = \int_0^{\pi}\frac{i\, d\theta}{2\pi} \qquad = \int_0^{\pi}\frac{I_m \, \mathrm{Sin}\theta\, d\theta}{2\pi}$$

$$= \frac{I_m}{2\pi}\int_0^{\pi}\mathrm{Sin}\theta\, d\theta \quad = \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{2\pi}\times 2$$

$$\mathbf{I_{av}} = \frac{\mathbf{I_m}}{\boldsymbol{\pi}}$$

14. Determine the form factor for the saw tooth wave from shown in Fig.



Solution : Let K be the slope of the portion OA. The instantaneous valu the wave form can be written as

$$e = k\,t = \frac{E_m}{T}t \quad \text{or} \quad e^2 = \frac{E_m^2\, t^2}{T^2}$$

The average of

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and peak fa

15. Determine t form shown

Solution :

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The average of the squared value can be written as

$$\text{Mean value of } e^2 = \frac{1}{T}\int_0^T \frac{E_m^2\, t^2}{T^2}\, dt = \frac{E_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T = \frac{E_m^2}{T^3}\left[\frac{T^3 - 0}{3}\right] = \frac{E_m^2}{3}$$

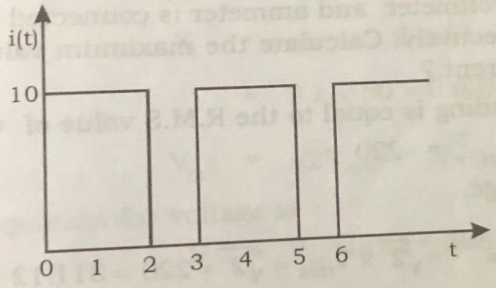
$\therefore$ The rms value, $E_{rms} = \sqrt{\frac{E_m^2}{3}} = \frac{E_m}{3}$

Also the average value, $= \frac{E_m + 0}{2} = \frac{E_m}{2}$

So, form factor $= \frac{E_{rms}}{E_{av}} = \frac{E_m/\sqrt{3}}{E_{av}/2} = \frac{2}{\sqrt{3}} = 1.155$

and peak factor $= \frac{E_{max}}{E_{rms}} = \frac{E_m}{E_m/\sqrt{3}} = \sqrt{3} = 1.732$

15. Determine the rms value, average value and form factor of the current wave form shown in fig.



Solution : The wave form is a periodic wave form with a period of 3 seconds.

$$I_{rms} = \sqrt{\frac{10^2 \times 2 + 0^2 \times 1}{3}} = 8.16 \text{ A}$$

$$I_{av} = \frac{10^2 \times 2 + 0^2 \times 1}{3} = 6.67 \text{ A}$$

Form factor $= \frac{I_{rms}}{I_{av}} = \frac{8.16}{6.67} = \mathbf{1.22}$

16. Find the r.m.s value of the resultant current in a wire carrying simultaneously a direct current of 10 A and a sinusoidal alternating current of peak value 10 A?

The resultant current at any instant is given by

$$i = 10 + 10 \sin \theta$$

Its r.m.s value is given by

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 10 \sin\theta)^2 \, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} (1 + \sin\theta)^2 \, d\theta} = \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1 + \sin^2\theta + 2\sin\theta) \, d\theta}$$

$$= \sqrt{\frac{50}{\pi}\left[\theta - 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}} = \sqrt{\frac{50}{\pi}(2\pi + \pi)}$$

$$= \sqrt{\frac{50}{\pi} \times 3\pi} = \sqrt{150} = \mathbf{12.247\ A.}$$

17. In an A.C Circuit, the voltmeter and ammeter is connected and shows 220 volts and 12 amps respectively. Calculate the maximum value and average value of voltage and current.?

Solution : Voltmeter reading is equal to the R.M.S value of voltage

$$V_{rms} = 220$$

Maximum value of voltage,

$$V_m = \sqrt{2} \times V_{rms} = \sqrt{2} \times 220 = 311.12 \text{ Volts}$$

Average value of voltage,

$$V_{av} = 0.637\, V_m = 0.637 \times 311.12 = 198.18 \text{ Volts}$$

Ammeter reading is equal to the R.M.S value of current

$$I_{rms} = 12$$

$$I_m = \sqrt{2}\,.\,I_{rms} = \sqrt{2} \times 12 = 16.97 \text{ A}$$

$$I_{av} = 0.637 \cdot I_m = 0.637 \times 16.97 = \mathbf{10.8\ A}$$

18. A 50 Hz, 230 Volt (rms) voltage is applied to a 1000 ohm resistor. Write equations for voltage and current as function of time 't'. Assume that at t = 0, v = 0 and going positive.

Solution :

$$V_{rms} = 230 \text{ Volts}, \ f = 50 \text{ Hz}, \ R = 1000\ \Omega$$

$$V_m = \sqrt{2}\ V_{rms} = \sqrt{2} \times 230 = 325.26 \text{ volt}$$

$$I_m = \frac{V_m}{R} = \frac{325.26}{1000} = 0.32526 \text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad/sec}$$

The equations are :

$$v = V_m \sin \omega t$$

$$v = 325.26 \sin (314 t)$$

$$i = I_m \sin \omega t$$

$$\mathbf{i = 0.32526 \sin (314\ t)}$$

19. A 50 Hz, 230 volt (rms) voltage is applied to a 0.637H inductor. Write the time equations for the applied voltage and current through the inductor. Assume that at t = 0, v = 0 and is going positive.

Solution :

$$V_{rms} = 230 \text{ volts}, \ f = 50 \text{ Hz}, \ L = 0.637 \text{ H}$$

$$X_L = \omega L = 2\pi f L$$

$$= 2\pi \times 50 \times 0.637 = 200\ \Omega$$

$$V_m = \sqrt{2} V_{rms} = \sqrt{2} \times 230 = 325.26 \text{ volt}$$

The time equation for voltage is

$$V = V_m \sin \omega t = 325.26 \sin (314\ t)$$

$$I_m = \frac{V_m}{X_L} = \frac{325.26}{200} = 1.626 \text{ A}$$

The time equation for current is

$$i = I_m \sin (\omega t - \pi/2) = \mathbf{1.626 \sin (314\ t - \pi/2)}$$

20. A 50 Hz, 230 volt (rms) voltage is applied to a capacitor of value 0.636 μF. Write equations for instantaneous voltage and current as functions of time 't'. Assume that at t = 0, V = 0 and going positive.

Solution :

$$V_{rms} = 230 \text{ volts}, \ f = 50 \text{ Hz}, \ C = 0.636 \times 10^{-6} \text{F}$$

$$V_m = \sqrt{2} V_{rms} = \sqrt{2} \times 230 = 326.26 \text{ volt}$$

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/sect

$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 0.636 \times 10^{-6}} = 5007.4\ \Omega$

$I_m = \frac{V_m}{X_C} = \frac{325.26}{5007.4} = 0.06469$ A

The equations are

$v = V_m \sin \omega t = 325.26 \sin(314\,t)$

$i = I_m \sin(\omega t + \pi/2) =$ **0.0649 sin (314 t + π/2)**

21. A 50 Hz sinusoidal voltage (40 +j30) volts is applied to a series R-L circuit resulting in sinusoidal current (4 + j1) amperes. Calculate (1) the impedance of the circuit (2) power consumed in the circuit and (3) power factor of the circuit.

Solution :

$V = 40 + j\,30 = 50 \angle 36.9^0$

$I = 4 + j\,1 = 4.1230 \angle 14^0$

Impedance, $Z = \frac{V}{I} = \frac{50 \angle 36.9^0}{4.123 \angle 14^0}$

$Z = 12.13 \angle 22.9^0$

$Z = 11.17 + j\,4.72$

Hence $R = 11.17\ \Omega$ and $X_L = 4.72\ \Omega$

Power consumed $= I^2R = (4.123)^2 \times 11.17 =$ **189.88 Watts**

Power factor $= \cos\phi = \cos 22.9^0 =$ **0.921**

22. A 230 V, 50 Hz supply is applied to a coil of resistance R = 10 Ω and inductance 0.2H. Calculate (i) the reactance and impedance of the coil. (ii) current and its phase angle relative to applied voltage.

$R = 10\ \Omega$, $L = 0.2$ H, $f = 50$ Hz

Reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.82\ \Omega$

Impedance, $Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 62.82^2} = 63.61\ \Omega$

Current, $I = \frac{V}{Z} = \frac{230}{63.61} = 3.615$ A

$\cos\phi = \frac{R}{Z} = \frac{10}{63.61} = 0.157$

Phase angle, $\phi = \cos^{-1}(0.157) =$ **81⁰**

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23. A 100 Ω resistor in series with 120 μ f capacitor is connect to 230 V, 50 Hz supply. Find (i) circuit impedance (ii) current (iii) power factor (iv) phase angle (v) voltage across R (vi) voltage across C.

Solution :

$R = 100\,\Omega,\ C = 120 \times 10^{-6}F,\ f = 50\ Hz$

Capacitive Reactance, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 26.5\,\Omega$

Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 26.5^2} = 103.45\,\Omega$

Current, $I = \frac{V}{Z} = \frac{230}{103.45} = 2.22\ A$

Power factor $\cos\phi = \frac{R}{Z} = \frac{100}{103.45} = 0.966$ leading

Phase angle, $\phi = \cos^{-1}(0.966) = 14.98^0$

Voltage across R, $V_R = IR = 2.22 \times 100 = 222$ Volt

Voltage across C, $V_C = IX_C = 2.22 \times 26.5 =$ **58.83 volt**

24. A resistor of resistance 10 Ω an inductance of 0.3 H and a capacitance of 100 μ F are connected in series across 230V, 50Hz mains. Calculate (i) Impedance (ii) Current (iii) Voltage across R, L & C (iv) Power in watts (v) Power factor.

$R = 10\,\Omega,\ L = 0.3\,H,\ C = 100\,\mu F,\ f = 50\ Hz$

$X_L = 2\pi f L = 2\pi \times 500.3 = 94.24\,\Omega$

Capacitive Reactance, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 31.83\,\Omega$

Impedance, $Z = \sqrt{R^2 + (X_L \times X_C)^2} = \sqrt{10^2 + (94.24 \times 31.83)^2} = 63.2\,\Omega$

Current, $I = \frac{V}{Z} = \frac{230}{63.2} = 3.64\ A$

Voltage across R, $V_R = IR = 3.64 \times 10 = 36.4$ Volt

Voltage across L, $V_L = IX_L = 3.64 \times 94.24 =$ 343 Volt

Voltage across C, $V_C = IX_C = 3.64 \times 31.83 =$ 115. 86 Volt

Power in watts $= VI\cos\phi = 230 \times 3.64 \times 0.158 = 132.27$ Watts

Power factor, $\cos\phi = \frac{R}{Z} = \frac{10}{63.2} =$ **0.158**

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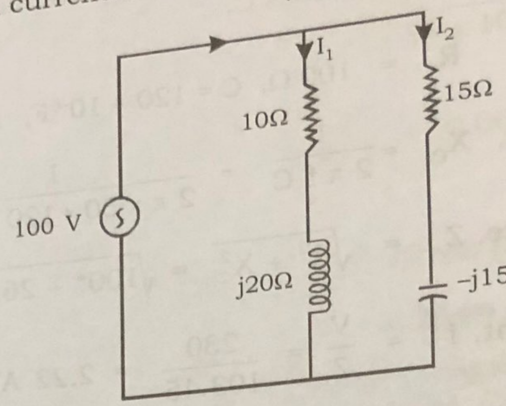
25. Find Z_{eq} for the parallel circuit shown in figure. Also find the total current and the branch currents.



Solution :

Let I denote the total current and let I_1 and I_2 denote the branch current. Branch I is a series combination of resistance 10 Ω and inductive reactance 20 Ω.

Its impedence, $Z_1 = (10 + j\,20)\ \Omega$

Branch 2 is a series combination of resistance 15 Ω and capacitive reactance 15Ω.

Its impedence, $Z_2 = (15 - j\,15)\ \Omega$

$$Z_{eq} = \frac{(Z_1 \cdot Z_2)}{(Z_1 + Z_2)} = \frac{[(10 + j20)(15 - j15)]}{[(10 + j20) + (15 - j15)]}$$

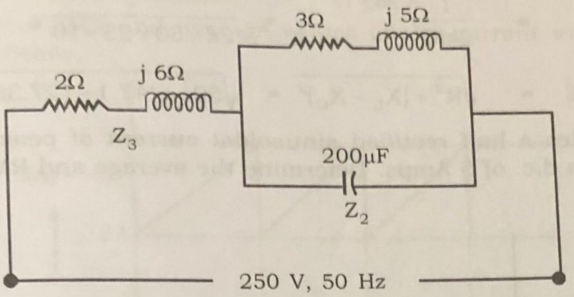
$$= \frac{[(22.36 \angle 63.43^0)\,(21.21 \angle -45^0)]}{[(25.495 \angle 11.3^0)\,]} = (18.6 \angle 7.13^0)\ \Omega$$

$$\text{Total current, } I = \frac{V}{Z_{eq}} = \frac{(100 \angle 0^0)}{(18.6 \angle 7.13^0)} = 5.37 \angle -7.13^0 \text{ ohms}$$

$$I_1 = \frac{V}{Z_1} = \frac{(100 \angle 0^0)}{(22.36 \angle 63.43^0)} = 4.472 \angle -63.43 \text{ A}$$

$$I_2 = \frac{V}{Z_2} = \frac{(100 \angle 0^0)}{(21.21 \angle -45^0)} = \mathbf{4.72 \angle 45\ A}$$

26. A voltage of 250V at 50Hz frequency is applied to the circuit shown in figure. Find (1) Current drawn from the source (2) power factor of the circuit (3) power consumed.



Solutions:

$Z_1 = (3 + j\,5)\ \Omega = 5.831 \angle 59.04^0\ \Omega$

$Z_2 = -j\,X_C = -j\,(1/2\pi f\,C) = -j\,(1/2\,\pi \times 50 \times 200 \times 10^{-6}) = -j\,15.915\ \Omega$

$Z_3 = (2 + j6)\ \Omega$

Equivalent impedance of parallel circuit

$$Z_{12} = \left(\frac{(Z_1 \cdot Z_2)}{(Z_1 + Z_2)}\right)$$

$Z_1 + Z_2 = (3 + j5) - j\,15.915 = (3 - j\,10.915)\ \Omega = \quad 11.32 \angle -74.68^0$

$$Z_{12} = \frac{[(5.831\angle 59.04^0)(15.915\angle -90^0)}{(11.32\angle -74.68^0)} = 8.1726 \angle 43.67^0$$

$= (5.9115 + j\,5.6432)\ \Omega$

Total impedance $Z = Z_{12} + Z_3 = (5.9115 + j\,5.6432) + (2 + j6)$

$= 7.9115 + j\,11.6432 = 14.0768 \angle 55.8^0\ \Omega$

$$\text{Current, } I = \frac{V}{Z} = \frac{(250\angle 0^0)}{(14.0678\angle 55.8^0)} = 17.76\angle -55.80^0 A$$

The magnitude of the current is 17.76 A and it lags behind the applied voltage by 55.8^0.

Power factor Cos ϕ = cos 55.8^0 = 0.5621

Total power, P = VI cos ϕ = 250 × 17.76 × 0.5621 = **2495.72 watts**

27. A series R–L–C circuit with a resistance of 50 ohms, a capacitance of 25 micro farad and an inductance of 0.15 Henry is connected across 230 volts, 50 Hz supply. Determine the impedance of the circuit?

Ans. R = 50 Ω, L = 0.15 H, C = 25 μF

X_L = $2\pi f L$ = $2\pi \times 50 \times 0.15$ = 47.1 Ω

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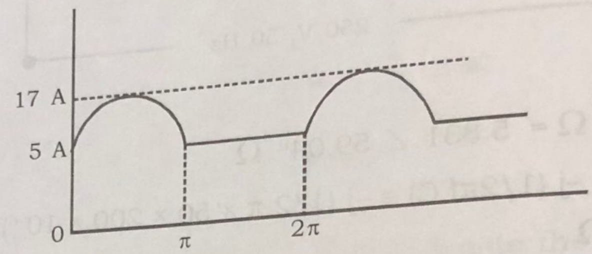
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = \mathbf{127.38}\ \Omega$$

$$\text{Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{50^2 + (47.1 - 127.38)^2} = 94.57\ \Omega$$

28. A resistance carries a half rectified sinusoidal current of peak value 12 Amps superimposed on a d.c. of 5 Amps. Determine the average and RMS values of the total current?



Ans.

$$I_{av} = \frac{1}{2\pi}\left[\int_0^{\pi}(5 + 12\sin\theta)d\theta + \int_{\pi}^{2\pi} 5 \cdot d\theta\right] = 8.819\text{ A}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}(5 + 12\sin\theta)^2 d\theta + \int_{\pi}^{2\pi} 5^2 \cdot d\theta\right]} = 9.959\text{ A}$$

29. Applied voltage to a series circuit is 100 sin (ωt + 10) and the current is 10sin(ωt – 30). Find the power?

Ans. v = 100 sin (ωt + 10), i = 10 sin (ωt – 30)

$$V_m = 100;\ V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}$$

$$I_m = 10,\ I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}}$$

Here the voltages leads the current by 40^0. So $\phi = 40^0$

$$\text{Power} = V_{rms} \times I_{rms} \times \cos\phi = \frac{100}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 40 = \mathbf{383\ W}$$

30. An alternating voltage of (80+ j60) volts applied to a circuit in which the current is (4+j10) amps. Find the impedance of the circuit, the power consumed and the phase angle?

Ans. $\overline{V} = 80 + j60,\quad \overline{I} = 4 + j\,10$

$$\text{Impedance, } Z = \frac{\overline{V}}{\overline{I}}$$

$$\text{Phase angle} = \cos^{-1}\frac{R}{Z}$$

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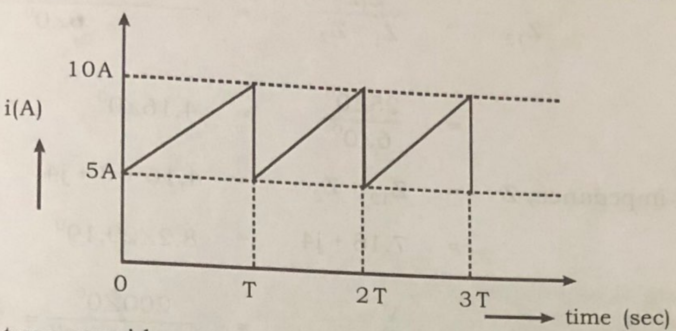
Power consumed $= VI \cos \phi$

31. Calculate the RMS and average values of the current wave form shown in the figure below.



Ans. Let us consider one repeat of duration T second.

Applying the equation of a straight line given by $y = mx + c$ (where m is the slope and c is the y-intercept)

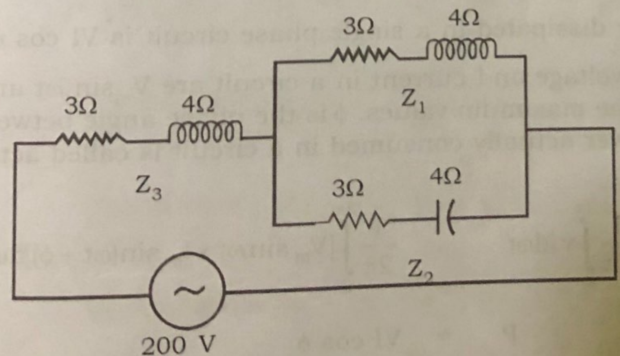
$$i = \left(\frac{5}{T}\right)t + 5$$

$$\text{Average value, } I_{av} = \frac{\text{Area under the waveform}}{\text{base}} = \frac{1}{T}\int_0^T i\,dt$$

$$= \frac{1}{T}\int_0^T \left[\frac{5}{T}t + 5\right]dt = 7.5 \text{ A}$$

$$\text{RMS value, } I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\frac{1}{T}\int_0^T \left[\frac{5}{T}t + 5\right]^2 dt} = 7.64 \text{ A}$$

32. Calculate the total current and the power factor of the following circuit.



Ans.

$$Z_1 = 3 + j4 = 5\angle 53.13^0$$

$$Z_2 = 3 - j4 = 5\angle -53.13^0$$

Z_1 and Z_2 are parallel. Equivalent impedance of the parallel circuit is given by

$$Z_{12} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{5\angle 53.13^0 \times 5\angle -53.13^0}{6\angle 0^0}$$

$$= \frac{25\angle 0^0}{6\angle 0^0} = 4.16\angle 0^0$$

$$\text{Total impedance, } Z = Z_{12} + Z_3 = 4.16 + 3 + j4$$

$$= 7.16 + j4 = 8.2\angle 29.19^0$$

$$\text{Current, } I = \frac{V}{Z} = \frac{200\angle 0^0}{8.2\angle 29.19^0} = 24.39\angle -29.19^0$$

The magnitude of the current is 24.39 a and it lags behind the applied voltage by 29.19^0.

Power factor, $\cos\phi$ = cos 29.19 = **0.873**

33. A resistance of 6Ω and inductance of 10 mH are connected in series, across 100 V 50 hz supply. Find its impedance.

Ans. R = 6 Ω, L = 10 mH = 10×10^{-3} H

$$X_L = 2\pi fL = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 3.14^2} = 6.77\ \Omega$$

34. What is zero power factor load and unity power factor load?

Ans. Zero power factor load : In this load the phase difference between current and voltage is 90^0. Zero power factor can be leading or lagging.

Unity power factor load : In this load the voltage and current are in phase. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time.

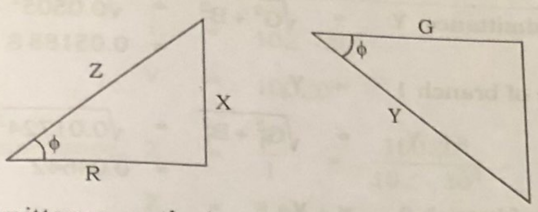
35. Prove that the power dissipated in a single phase circuit is VI cos ϕ?

Ans. Consider the voltage and current in a circuit are $V_m \sin \omega t$ and $I_m \sin(\omega t + \phi)$. Here V_m and I_m are the maximum values. ϕ is the phase angle between the voltage and current. The power actually consumed in a circuit is called active power.

$$\text{Active power, } P = \frac{1}{2\pi}\int_0^{2\pi} vi\,d\omega t = \frac{1}{2\pi}\int_0^{2\pi} [V_m \sin\omega t \times I_m \sin(\omega t + \phi)]d\omega t$$

$$P = VI\cos\phi$$

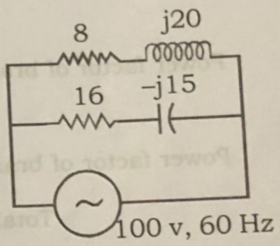
36. Two impedence (8+j20) ohm and (16 – j 15) are connected in parallel across 100 V 60 Hz supply. Find the currents in each branch, power factor of each branch and total power dissipated in the whole circuit by admittance method?



Ans. Admittance method.

Admittance, Y $= \frac{1}{Z}$

As impedance (Z) has two rectangular components R and X. Similarly, admittance Y also has two components. The X-component is called conductance (G) and the Y component is known as susceptance (B). The unit of admittance, conductance and susceptance is Siesmens (S).



Conductance (G) $= Y\cos\phi$

$= \frac{1}{Z}\cdot\frac{R}{Z}$

$= \frac{R}{Z^2}$

$= \frac{R}{R^2+X^2}$

Susceptance (B) $= Y\sin\phi = \frac{1}{Z}\cdot\frac{X}{Z} = \frac{X}{R^2+X^2}$

Admittance (Y) $= \sqrt{G^2+B^2}$

While representing admittance, capacitive susceptance should be taken as positive and inductive susceptance as negative.

Conductance, $G_1 = \frac{R_1}{R_1^2+X_1^2} = \frac{8}{8^2+20^2} = 0.01724$ S

Conductance, $G_2 = \frac{R_2}{R_2^2+X_2^2} = \frac{16}{16^2+15^2} = 0.03326$ S

Susceptance, $B_1 = \frac{-X_1}{R_1^2+X_1^2} = \frac{-20}{8^2+20^2} = -0.0431$ S

Susceptance, $B_2 = \frac{X_2}{R_2^2+X_2^2} = \frac{15}{16^2+15^2} = 0.03118$ S

Total conductance G $= G_1 + G_2 = 0.01724 + 0.03326$

$= 0.0505$ S

Total susceptance, B $= B_1 + B_2 = -0.0431 + 0.03118$

$= -0.01192$ S

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$$\text{Total admittance, } Y = \sqrt{G^2 + B^2} = \sqrt{0.0505^2 + -0.01192^2} = 0.05188 \text{ S}$$

$$\text{Admittance of branch 1} = Y_1$$

$$Y_1 = \sqrt{G_1^2 + B_1^2} = \sqrt{0.01724^2 + -0.0431^2} = 0.04642$$

$$\text{Admittance of branch 2} = Y_2$$

$$Y_2 = \sqrt{G_2^2 + B_2^2} = \sqrt{0.03326^2 + 0.03118^2} = 0.04558$$

$$\text{Current through branch 1 } (I_1) = VY_1 = 100 \times 0.04642 = 4.642 \text{ A}$$

$$\text{Current through branch 2} (I_2) = VY_2 = 100 \times 0.04558 = 4.558 \text{ A}$$

$$\text{Power factor of branch 1 } (\cos\phi_1) = \frac{G_1}{Y_1} = \frac{0.01724}{0.04642} = 0.371$$

$$\text{Power factor of branch 2 } (\cos\phi_2) = \frac{G_2}{Y_2} = \frac{0.03326}{0.04558} = 0.729$$

$$\text{Total current, } I = VY = 100 \times 0.05188 = 5.188 \text{ A}$$

$$\text{Power factor } (\cos\phi) = \frac{G}{Y} = \frac{0.0505}{0.05188} = 0.973$$

$$\text{Total power} = VI\cos\phi = 100 \times 5.188 \times 0.973 = \textbf{504.79 Watts}$$

37. A coil of power factor 0.6 in series with a 100 μF capacitor. When it is connected a 50 Hz supply the potential difference across the coil is equal to the potential difference across the capacitor. Find the resistance and inductance of the coil.

Ans.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}} = 31.8\ \Omega$$

Since p.d. across the coil is equal to the p.d. across the capacitor, it means that:

$$\text{Coil impedance} = X_C$$

$$\therefore \text{Coil } Z = 31.8\ \Omega$$

$$\text{Coil p.f} = \cos\phi = 0.6$$

$$\sin\phi = 0.8$$

$$\text{Coil } R = Z\cos\phi = 31.8 \times 0.6 = 19.08\ \Omega$$

$$\text{Coil } X = Z\sin\phi = 31.8 \times 0.8 = 25.44\ \Omega$$

38. A current of 10 A flows in a circuit with a 30° angle of lag, when applied voltage 100 V. Find (i) the resistance, reactance and impedance, (ii) conductance susceptance and admittance?

Ans.

$$I = 10\angle -30^0$$

$$V = 100\angle 0^0$$

$$\text{Impedance, } Z = \frac{V}{I} = \frac{100\angle 0^0}{10\angle -30^0} = 10\angle 30^0$$

$$Z = 8.66 + j5$$

$$\text{Resistance} = 8.66\ \Omega$$

$$\text{Reactance} = 5\ \Omega$$

$$\text{Admittance, } Y = \frac{1}{Z} = \frac{1}{10\angle 30^0} = 0.1\angle -30^0$$

$$Y = 0.0866 - j\,0.05$$

$$\text{Conductance (G)} = 0.0866$$

$$\text{Susceptance (B)} = 0.05$$

39. The current in a circuit is given by

$$i = 100 \sin 728\, t$$

Find the maximum value and frequency of the current?

Ans.

$$i = I_m \sin \omega t$$

$$\text{Maximum value of current, } I_m = 100\text{ A}$$

$$\omega t = 728\, t$$

$$\therefore\ \omega = 728$$

$$2\pi f = 728$$

$$\text{Frequency, } f = 115.9\text{ Hz}$$

40. The apparent power drawn by an a.c. circuit is 10 kVA and the active power is 8 kW. what is the reactive power in the circuit? What is the power factor of the circuit.

Ans.

$$\text{Power factor} = \frac{\text{Active power}}{\text{Apparent power}} = \frac{8}{10} = 0.8$$

$$\text{Cos}\ \phi = 0.8,\ \sin\phi = 0.6$$

$$\text{Reactive power} = VI \sin\phi = 10 \times 0.6 = \mathbf{6\ kVAR}$$

41. The impedance offered by an a.c. circuit is (8 + j6) ohms. Find the admittance of the circuit in symbolic form?

Ans.

$$Z = 8 + j6 = 10\angle 36.86^0$$

$$\text{Admittance, } Y = \frac{1}{Z} = \frac{1}{10\angle 36.86^0} = 0.1\angle -36.86^0$$

$$= 0.08 - j\,0.059$$

42. In a saw tooth wave form, the current increases linearly from 0 to 2 A and then suddenly drops to zero as represented below. Find the average and r.m.s. values of the current.

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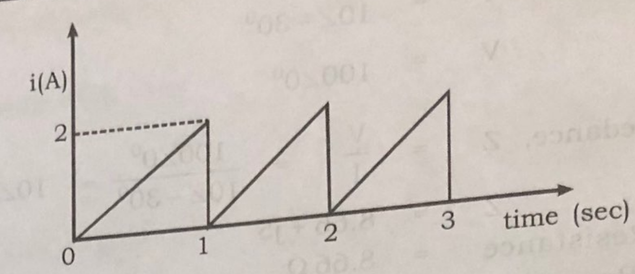


Ans.

$$I_{av} = \frac{\text{Area}}{\text{Base}} = \frac{1/2 \times 1 \times 2}{1} = 1\text{ A}$$

$$\text{Slope} = 2/1 = 2$$

Applying the equation $y = mx$

i.e., $i = 2t$

$$I_{rms}^2 = \frac{1}{1}\int_0^1 i^2 dt = \int_0^1 (2t)^2 dt$$

$$= \int_0^1 4t^2 dt = 4\left[\frac{t^3}{3}\right]_0^1 = 4 \times \frac{1}{3}$$

$$\therefore\ I_{rms} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}\text{ ampere}$$

43. 2. The r.m.s. value of a right-angled triangular wave is 200 volts. What will its peak voltage?

Ans. $V_{rms} = 200$ volts

$$V_{rms} = \frac{V_m}{\sqrt{3}}$$

$$\text{Peak voltage, } V_m = \sqrt{3} \times V_{rms} = \sqrt{3} \times 200 = 346.4\text{ volts.}$$

44. A sine wave of voltage varies from zero to maximum value of 200 volts. What is the magnitude of the voltage at the instant of 60^0 of the cycle?

Ans.

$$v = V_m \sin\theta$$

$$v = 200 \times \sin 60^0 = 200 \times 0.866 = 173.2\text{ volts}$$

Magnitude of the voltage at the instant of 60^0 = 173.2 volts

45. An alternating voltage V = (160 + j120) volts is applied to a circuit and the current flowing I = –6 + j 15 amps. Find the impedance and power factor of the circuit?

Ans.

$$V = 160 + j\,120 = 200\angle 36.86^0$$

$$I = -6 + j\,15 = 16.16\angle 111.8^0$$

$$\text{Impedance, } Z = \frac{V}{I} = \frac{200\angle 36.86^0}{16.16\angle 111.8^0} = 12.37\angle -74.94^0$$

$$\text{Power factor, } \cos\phi = \cos 74.94^0 = 0.26$$

46. The impedance in rectangular form is (8− j 6) Ohms. What is its admittance in polar form?

Ans.

$$Z = 8 - j6 = 10\angle -36.86^0$$

$$\text{Admittance, } Y = \frac{1}{Z} = 0.1\angle 36.86^0$$

47. A circuit takes a current of 3 Ampere at a p.f of 0.6 lagging when connect to a 150volt, 50 Hz supply. Another circuit takes a current of 5A at a p.f. of 0.8 leading when connected to the same supply. If these two circuits are connected in series across 230 volts, 50Hz Supply, Calculate the current drawn from the supply?

Ans. First Circuit

I_1 = 3A at a p.f. of 0.6 lagging = $3\angle -53.13^0$

$$V_1 = 150\angle 0^0$$

$$Z_1 = \frac{V_1}{I_1} = \frac{150\angle 0^0}{3\angle -53.13^0} = 50\angle 53.13^0 = 30 + j\,40$$

I_2 = 5 A at a p.f. of 0.8 leading = $5\angle 36.86^0$

$$V_2 = 150\angle 0^0$$

$$Z_2 = \frac{V_2}{I_2} = \frac{150\angle 0^0}{5\angle 36.86^0} = 30\angle -36.86^0$$

$$= 24 - j\,18$$

The above two circuits are connected in series across 230 volts supply, Then the equivalent impedance is given by

$$Z = Z_1 + Z_2 = 54 + j\,22 = 58.3\angle 22.16^0$$

$$V = 230\angle 0^0$$

$$I = \frac{V}{Z} = \frac{230\angle 0^0}{58.3\angle 22.16^0} = 3.9\angle -22.16^0\,A$$

48. The r.m.s. value of a half wave rectified current is 100 A. What would be its r.m.s value for full wave rectification?

Ans.

$$I_{rms} = 100\text{ A}$$

Rms value of a half wave rectied signal = $I_M/2$

$$I_{rms} = I_M / 2$$

From the equation

$$I_M = 100 \times 2 = 200\text{ A}$$

rms value for full wave rectified circuit, $I_{rms} = \frac{I_M}{\sqrt{2}} = \frac{200}{\sqrt{2}}$ = **141.42**

49. An A.C. current is given by $i = 200 \sin 100\,\pi t$. Find the time to reach a value of 100 amps before reaching its final value.

Ans. Instantaneous value $i = 200 \sin 100\pi t$

$$100 = 200 \sin (100 \times 180 \times t)$$

$$\frac{100}{200} = \sin (100 \times 180 \times t)$$

$$\sin^{-1}(0.5) = 18000\,t$$

$$t = \frac{\sin^{-1}(0.5)}{18{,}000} = \frac{1}{600} \text{ sec.}$$

50. A single-phase RC series circuit consists of 8 ohms resistance and 100 μF. A voltage of 200 volts at 50 Hz is applied to the circuit. What is the impedance offered by the circuit?

Ans. $R = 8\ \Omega,\quad C = 100 \times 10^{-6}$ F

Applied voltage $V = 200$ V

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + X_C{}^2} = 32.81\ \Omega$$

Or

$$Z = R - j \times C = 8 - j31.83 = 32.81\angle -75.9\ \Omega$$

51. An impedance is expressed in rectangular form as (16 – j12) ohms. Express the impedance value in polar form.

Ans.

$$Z = \sqrt{16^2 + (12)^2} = 20\ \Omega$$

$$\text{Angle} = \tan^{-1}\left(\frac{-12}{16}\right) = -36.86$$

So, 'Z' in polar form $= 20\angle -36.86\ \Omega$

52. A series circuit consists of R = 20 ohms, L = 20 mH and an a.c. supply of 60V with f = 100 Hz. Find the voltage across L.

Ans. $R = 20\ \Omega,\quad L = 20 \times 10^{-3}$ H

Applied voltage $V = 60$ V

Supply frequency, $f = 100$ Hz

$$X_L = 2\pi fL = 2\pi \times 100 \times 20 \times 10^{-3} = 12.56\ \Omega$$

$$\text{Impedance } Z = (20 + j12.56) = 23.61\angle 32.12\ \Omega$$

Then, current through the circuit $= \dfrac{60\angle 0}{23.61\angle 32.12} = 2.54\angle -32.12$ A

Voltage drop across inductor, $V_L = 2.54 \times 12.56 = 31.9$ V

53. Perform the following operations and express the result in the polar form

(i) $(10 - j10)\cdot(10\angle -60^0)(10e^{j30^0})$

Ans. Convert (10 – j10) into polar form

$$\text{Magnitude} = \sqrt{10^2 + (-10)^2} = 14.14$$

$$\text{Angle} = \tan^{-1}\left(\frac{-10}{10}\right) = -45^0$$

$$\text{Polar form} = 14.14\angle -45^0$$

Convert 10 e^{j30} (exponential form to polar form) $= 10\angle 30$

$$\therefore \quad (14.14\angle -45) \times (10\angle -60) \times (10\angle 30) = 1414\angle -75^0$$

(ii) $\dfrac{(5e^{j30^0})\cdot(2 - j4)}{(3 + j^3)(2\angle -15^0)}$

Ans. Convert $5e^{j30}$ (Exponential form in to polar form) $= 5\angle 30^0$

$$(2 - j4) = 4.47\angle -63.4$$

$$(3 + j^3) = (3 - j1) = 3.1622\angle -18.43$$

Then $\dfrac{(5\angle 30)(4.47\angle -63.4)}{(3.1622\angle -18.43)(2\angle -15)} = \dfrac{22.36\angle -33.43}{6.32\angle -33.43} = 3.535\angle 0$

54. A coil connected to a 250 volts, 50 Hz sinusoidal supply takes a current of 10 A at a phase angle of 30^0 lagging. Calculate (i) the resistance and inductance of the coil and (ii) the power consumed by the coil.

Ans. Circuit impedance $Z = \dfrac{250\angle 0}{10\angle -30} = 25\angle 30\ \Omega = 21.65 + j12.5\ \Omega$

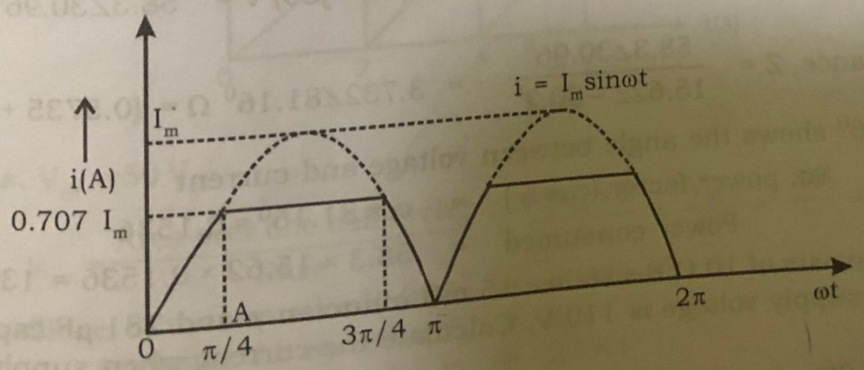
Resistance $R = 21.65\ \Omega$

& Reactance $X_L = 12.5\ \Omega$

Inductance of the coil $L = \dfrac{X_L}{2\pi f} = \dfrac{12.5}{2 \times \pi \times 50} = 0.0397$ H

Power consumed by the coil $= 250 \times 10 \times \cos 30 = 2.165$ kW

55. Determine the r.m.s. value of the current waveform shown below.



Ans. A full wave rectified sine function is clipped at 0.707 of its maximum value is shown in figure.

To find ωt at the point A,

$$I_m \sin\omega t = 0.707\, I_m$$

$$\therefore \quad \sin\omega t = 0.707$$

$$\omega t = \frac{\pi}{4}$$

$$\text{Time period} = \pi$$

$$i = I_m \sin\omega t \rightarrow 0 < \omega t < \frac{\pi}{4}$$

$$i = 0.707\, I_m \rightarrow \frac{\pi}{4} < \omega t < \frac{3\pi}{4}$$

$$i = I_m \sin\omega t \rightarrow \frac{3\pi}{4} < \omega t < \pi$$

Mean Square Value

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} (I_m \sin\omega t)^2 \cdot d\omega t + \int_{\pi/4}^{3\pi/4} (0.707\, I_m)^2 \cdot d\omega t + \int_{3\pi/4}^{\pi} (I_m \sin\omega t)^2 \cdot d\omega t\right]$$

$$= \frac{1}{\pi}\left[\frac{I_m^2}{8}(\pi - 2) + (0.707\, I_m)^2 \cdot \frac{\pi}{2} + \frac{I_m^2}{8}(\pi - 2)\right]$$

$$= \frac{1}{\pi}\left[2 \times \frac{I_m^2}{8}(\pi - 2) + (0.707\, I_m)^2 \cdot \frac{\pi}{2}\right] = 0.3407\, I_m^2$$

$$I_{rms} = \sqrt{0.3407 I_m^2} = 0.583\, I_m$$

56. The current in a circuit is (10 – j12) amperes when the applied voltage is (50 + j30) volts. Find (a) impedance of the circuit (b) power consumed and (c) power factor.

Ans.

$$\text{Current (I)} = (10 - j12)\text{ A} = 15.62\angle -50.2^0$$

$$\text{Supply voltage (V)} = (50 + j30)\text{ V} = 58.3\angle 30.96^0\text{ V}$$

$$\text{Impedance } Z = \frac{58.3\angle 30.96^0}{15.62\angle -50.2^0} = 3.732\angle 81.16^0\ \Omega = (0.5735 + j\,3.688)\ \Omega$$

Angle 81.16^0 shows the angle between voltage and current

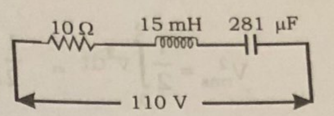
$$\text{So, power factor } (\cos\phi) = \cos 81.16^0 = 0.1536$$

$$\text{Power consumed} = 58.3 \times 15.62 \times 0.1536 = 139.87\text{ Watts}$$

57. A circuit consists of 10 Ω resistance 15 mH inductance and 281 μF capacitance in series. The supply voltage is 110 V. Calculate the current when supply frequency is 150 Hz.

Ans. $R = 10\Omega;\ L = 15 \times 10^{-3}\text{ H};\ C = 281 \times 10^{-6}\text{ F}$

Impedance of the circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$



$X_L = 2\pi fL = 2 \times \pi \times 150 \times 15 \times 10^{-3} = 14.13\ \Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 150 \times 281 \times 10^{-6}} = 3.77\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 14.39\ \Omega$

Current through the circuit $= \frac{110}{14.39} = 7.64$ A

58. A resistance of 20 Ω, and inductance of 0.2 H and a capacitance of 100 μF are connected in series across 220 V, 50 Hz supply. Determine the following: (a) Impedance; (b) Current; (c) Voltage across R, L and C; and (d) power factor.

Ans. $R = 20\ \Omega$; $L = 0.2$ H; $C = 100 \times 10^{-6}$ F

$X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{(2\pi \times 50 \times 100 \times 10^{-6})} = 31.83\ \Omega$

$Z = 20 + j\,(62.83 - 31.83) = 36.9\angle 57.17\ \Omega$

Power factor $(\cos\phi) = \cos(57.17) = 0.542$

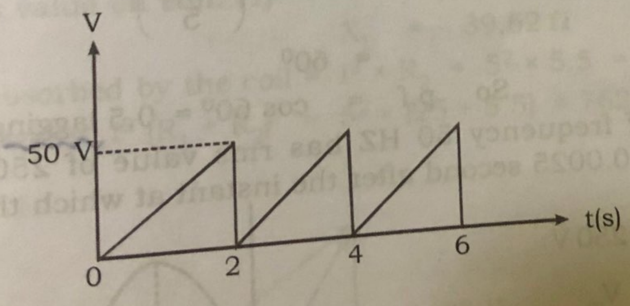
Current $I = \frac{220\angle 0}{36.9\angle 57.17} = 5.96\angle -57.15$ A

$V_R = 5.96 \times 20 = 119.2$ V

$V_L = 5.96 \times 62.83 = 374.46$ V

$V_C = 5.96 \times 31.83 = 189.7$ V

59. Find the average and r.m.s. values of the sawtooth waveform shown below:



Ans. $V_m = 50$ V

$E_{av} = \frac{\text{Area over one half cycle}}{\text{Base}} = \frac{\frac{1}{2}V_m \times 2}{2} = \frac{V_m}{2} = \frac{50}{2} = 25$ volts

Slope of the equation $(\tan\theta) = 50/2 = 25$

Applying the equation $Y = mx$

$v = 25 \times t$

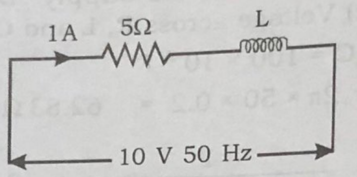
$$V_{rms}^2 = \frac{1}{2}\int_0^2 v^2 dt = \frac{1}{2}\int_0^2 (25t)^2 dt = 312.5\int_0^2 t^2 dt = 312.5\left[\frac{t^3}{3}\right]_0^2 = 312.5\left[\frac{2^3}{3} - 0\right]$$

$$= \frac{2500}{3}$$

$$V_{rms} = \sqrt{\frac{2500}{3}} = 28.86 \text{ volts}$$

60. A series circuit with R and L draws a current of 1A when connected across a 10 V, 50 Hz supply. Assuming the resistance to be 5Ω, find the inductance of the circuit. What is its power factor? Draw the phasor diagram of the circuit.

Ans.



Applied voltage $V = 10\text{ V}$

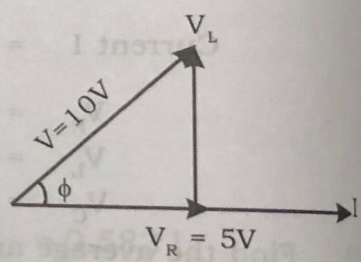
Voltage across resistance, $R = 1 \times 5 = 5\text{V}$

$$V_L = \sqrt{10^2 - 5^2} = 8.66\text{ V}$$

$$X_L = \frac{8.66}{1} = 8.66\ \Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{XL}{2\pi f} = \frac{8.66}{2 \times \pi \times 50} = 0.0275\text{ H}$$



Angle between voltage & current $\phi = \tan^{-1}\left(\frac{8.66}{5}\right)$

$$= 60^0$$

So $\text{p.f} = \cos 60^0 = 0.5$ lagging

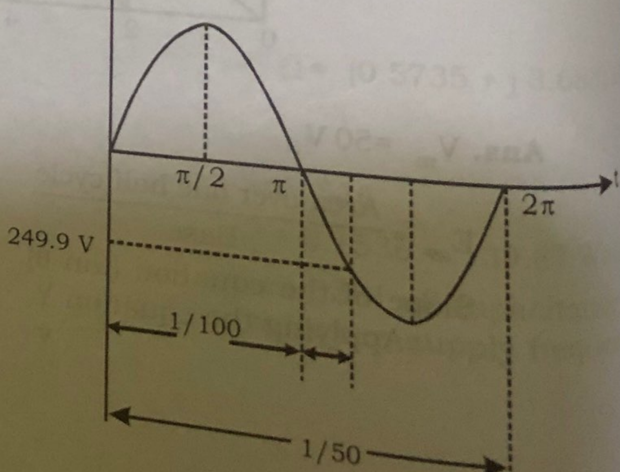
61. A sinusoidal emf of frequency 50 HZ has rms value of 250V. Calculate it's instantaneous value 0.0025 second after the instant at which the emf is zero and then decreasing.

Ans. $V_{rms} = 250\text{ V}$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_m = 250 \times \sqrt{2}$$

$$= 353.5\text{ V}$$



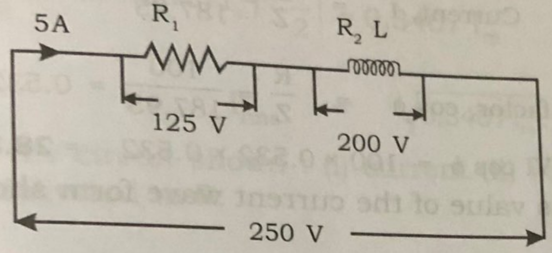
Total time $t = \frac{1}{100} + 0.0025$

$= 0.0125$ sec

$V = V_m \sin \omega t = V_m \sin \times 2\pi \times f \times t = 353.5 \times \sin 2 \times \pi \times 50 \times 0.0125 = -$ **249.96 V**

62. **A current of 5 A flows through a non-inductive resistance in series with a choking coil when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate (a) impedance, reactance and resistance of the coil. (b) The power absorbed by the coil and (c) The total power. Draw the vector diagram.**

Ans.



$$R_1 = \frac{125}{5} = 25\ \Omega$$

(a) Impedance of the coil (Z_2) $= \frac{200}{5} = 40\ \Omega$

$$R_2^2 + X_L^2 = 40^2 \quad \text{.................(1)}$$

Impedance of the circuit $Z = \frac{250}{5} = 50\ \Omega$

$$(R_1 + R_2)^2 + X_L^2 = 50^2$$

$$(25 + R_2)^2 + X_L^2 = 2500 \quad \text{.........(2)}$$

Subtracting eqn (1) from eqn. (2)

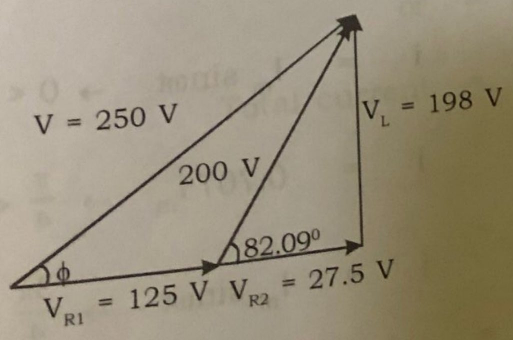
We get, $R_2 = 5.5\ \Omega$

Put this value on eqn. (1)

We get, $X_L = 39.62\ \Omega$

(b) Power absorbed by the coil $= I^2 \times R_2 = 5^2 \times 5.5 = 137.5$ W

(c) Total power $= I^2 (R_1 + R_2) = 5^2 \times (25 + 5.5) = 762.5$ W



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63. A resistor of 100 Ω and a capacitor of 20 μF are connected in series and the combination is connectd across a 100 V, 50 Hz supply. Find the power consumed and the power factor.

Ans. $R = 100\ \Omega$, $C = 20\ \mu F = 20 \times 10^{-6}$ F, $f = 50$ Hz

Capacitive Reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}} = 159.15\ \Omega$

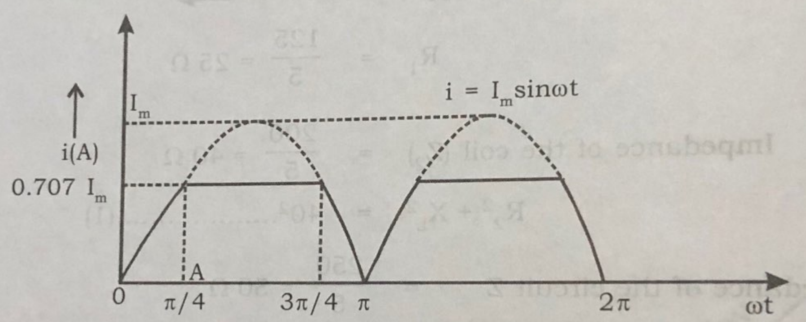
Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 159.15^2} = 187.95\ \Omega$

Current, $I = \frac{V}{Z} = \frac{100}{187.95} = 0.532$ A

Power factor, $\cos\phi = \frac{R}{Z} = \frac{100}{187.95} = 0.532$

Power consumed = $VI\cos\phi = 100 \times 0.532 \times 0.532$ = **28.3 Watts**

64. Determine the rms value of the current wave form shown below:



Ans. A full wave rectified sine function is clipped at 0.707 of its maximum value is shown in figure.

To find ωt at the point a,

$$I_m \sin\omega t = 0.707\, I_m$$

$$\therefore \sin\omega t = 0.707$$

$$\omega t = \frac{\pi}{4}$$

$$\text{Time period} = \pi$$

$$i = I_m \sin\omega t \rightarrow 0 < \omega t < \frac{\pi}{4}$$

$$i = 0.707\, I_m \rightarrow \frac{\pi}{4} < \omega t < \frac{3\pi}{4}$$

$$i = I_m \sin\omega t \rightarrow \frac{3\pi}{4} < \omega t < \pi$$

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Mean Square Value

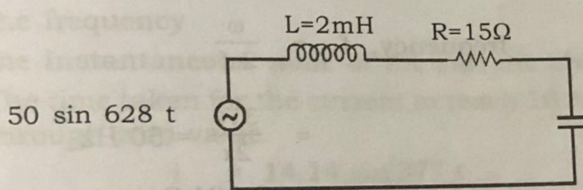
$$= \frac{1}{\pi}\left[\int_0^{\pi/4}(I_m \sin\omega t)^2 \cdot d\omega t + \int_{\pi/4}^{3\pi/4}(0.707\, I_m)^2 \cdot d\omega t + \int_{3\pi/4}^{\pi}(I_m \sin\omega t)^2 \cdot d\omega t\right]$$

$$= \frac{1}{\pi}\left[\frac{I_m^2}{8}(\pi-2) + (0.707\, I_m)^2 \cdot \frac{\pi}{2} + \frac{I_m^2}{8}(\pi-2)\right]$$

$$= \frac{1}{\pi}\left[2\times\frac{I_m^2}{8}(\pi-2) + (0.707\, I_m)^2 \cdot \frac{\pi}{2}\right] = 0.3407\, I_m^{\ 2}$$

$$I_{rms} = \sqrt{0.3407 I_m^2} = 0.583\, I_m$$

65. Determine, for the circuit shown : (i) current (ii) voltages across L and C, (iii) power delivered to R.



Ans.

$$\omega = 628 \text{ rad/sec}$$

$$X_L = \omega \times L$$

$$= 628 \times 2 \times 10^{-3} = 1.25\ \Omega$$

and $$X_C = \frac{1}{\omega \times C}$$

$$= \frac{1}{628 \times 1 \times 10^{-6}} = 1592\ \Omega$$

Then, $$X = X_L - X_C$$

$$= 1.25 - 1592 = -1590.75\ \Omega$$

Then $$Z = 15 - j\,1590.75$$

or $$= 1590 \angle -89.45$$

Total current $$= \frac{V}{Z}$$

$$= \frac{\frac{50}{\sqrt{2}}\angle 0}{1590\angle -89.45}$$

$$= 0.022 \angle 89.45$$

Voltage across inductance L $= I \times X_L$

$= 0.022 \times 1.25$

$= 0.0275$ V

Voltage across capacitor C $= I \times X_C$

$= 0.022 \times 1592$

$= 35.02$ V

Then power delivered to R is $= 0.022^2 \times 15$

$= 7.26$ mW

66. An emf is given by 170 sin 314 t V. Determine (i) maximum value (ii) rms value (iii) frequency (iv) radian through which its vector has gone, when t = 0.001 S and (v) value of emf at instant in (iv)

An.

$$v = 170 \sin 314\, t$$

$$V_m = 170 \text{ V}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{170}{\sqrt{2}} = 120.2 \text{ V}$$

$$\text{frequency, } f = \frac{\omega}{2\pi f}$$

$$= \frac{314}{2\pi} = 50 \text{ Hz}$$

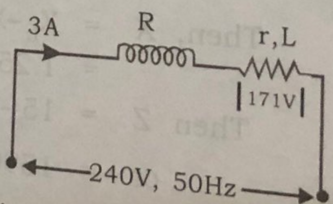
When $t = 0.001$ S,

$$v = 170 \times \sin(314 \times 0.001)$$

$$= 52.5 \text{ V}$$

This emf occurs at $\frac{\pi}{10}$ radian.

67. When a resistor and an inductor in series are connected to 240 V, 50 Hz a.c mains, a current of 3A flows lagging 37^0 behind the supply voltage, while the voltage across the inductor is 171 V. Determine the resistance of the resistor and resistance and inductance of the inductor. Draw the phasor diagram of the circuit.



Ans.

Circuit current, $I = 3\angle -37^0$ A

Applied voltage, $V = 240\angle 0^0$ V

Frequency, $f = 50$ Hz

Inductance, $Z = \frac{V}{I}$

$$= \frac{240\angle 0^0}{3\angle -37^0}$$

$= (63.9 + j48.14)\ \Omega$

Reactance of the coil $= 48.14$

$I X_L = 3 \times 48.14 = 144.42$ V

$\sin\phi = \frac{144.42}{171} = 0.844$

$\therefore \phi = 57.62^0$

$I \times r = \cos 57.62 \times 171 = 91.57$ V

$I(R + r) = \cos 37 \times 240 = 191.67$ V

So $IR = 191.67 - 91.57 = 100$ V

$\therefore R = \frac{100}{3} = 33.33\ \Omega$

$r = \frac{91.57}{3} = 30.52\ \Omega$

68. An alternating current is given by $i = 14.14 \sin 377\, t$. Find
   (i) the r.m.s value of the current
   (ii) the frequency
   (iii) the Instantaneous value of the current when it is 3 ms.
   (iv) The time taken for the current to reach 10 A for the first time after passing through zero value.

Ans. $i = 14.14 \sin 377\, t$

(i) $I_{rms} = \frac{14.4}{\sqrt{2}} = 9.9$ A

(ii) $\omega = 2\pi f = 377$

then $f = 60$ Hz

(iii) After time 3 m sec., $i = 14.14 \sin (2\pi \times 60 \times 3 \times 10^{-3} = 12.8$ A

(iv) $10 = 14.14 \sin (2 \times 180 \times 60 \times t)$

$\frac{10}{14.14} = \sin (120 \times 180 \times t)$

$\sin^{-1}\left(\frac{10}{14.14}\right) = 120 \times 180 \times t$

Then $t = \frac{1}{480}$ sec.

OR

69. A series circuit consists of a 300 Ω non inductive resistor, a 7.95 μF capacitor and a 2.06 H inductor of negligible resistance. If the supply voltage is 250 V at 50 Hz. Calculate
   (i) the circuit current
   (ii) the phase angle
   (iii) the voltage drop across each element

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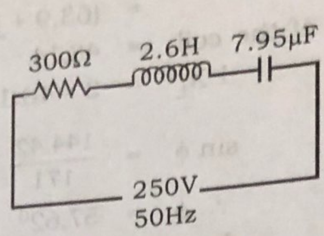
Ans.



$$X_L = 2\pi \times 50 \times 2.06 = 647.16\ \Omega$$

$$X_C = \frac{1}{2\pi \times 50 \times 7.95 \times 10^{-6}} = 400.3\ \Omega$$

Then impedance $Z = 300 + j\,(647.16 - 400.3)\ \Omega$

$= (300 + j\,246.86)\ \Omega$

$= 388.5 \angle 39.4\ \Omega$

70. A coil of insulated wire of resistance 8Ω and inductance 0.03 H is connected to an a.c. supply at 240V, 50Hz. Calculate:

(i) The current, p.f. and the power.

(ii) The value of capacitance which when connected in series with the above coil causes no change in the values of the current and power taken from the supply.

Ans. R = 8 Ω L = 0.03 H

Inductive reactance $X_L = 2\pi fL = 2\pi \times 50 \times 0.03 = 9.42\ \Omega$

Impedance $Z = \sqrt{8^2 + 9.42^2} = 12.36\ \Omega$

Current $I = \frac{240}{12.36} = 19.41$ A

Power factor $= \frac{R}{Z} = \frac{8}{12.36} = 0.648$ lagging

Power $P = 240 \times 19.41 \times 0.647 = 3.01$ kW

A capacitor is connected in series with this combination without changing circuit current and power. This will occur in leading p.f. 0.647 and at same impedance

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$X_C = 18.84\ \Omega$$

Value of capacitance $C = \frac{1}{2\pi f \times X_C} = \frac{1}{2\pi \times 50 \times 18.84} = 168.95\ \mu F$

71. An inductive coil takes 10A and dissipates 1000W when connected to a 250V, 25 Hz supply. Calculate to the following.
(i) the impedance, (ii) the effective resistance, (iii) the resistance, (iv) the power factor, (v) the value of the capacitance required to be connected in series with the coil to make the power factor of the circuit unity, what is now the current taken by the coil?

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Ans. Power dissipation = 1000 W

V = 250 V, f = 25 Hz, I = 10 A

(i) $Z = \frac{250}{10} = 25\ \Omega$

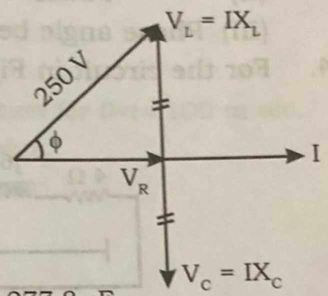
$\cos\phi = \frac{1000}{250 \times 10} = 0.4$

Phase angle $\phi = \cos^{-1} 0.4 = 66.42$

Impedance in polar form $= 25\angle 66.42 = 10 + j22.91\ \Omega$

Resistance $= 10\ \Omega$

(v) In order to make the power factor unity



$IX_L = IX_C$

and $X_L = X_C$

$X_C = 22.91\ \Omega$

Capacitance required $C = \frac{1}{2\pi f X_C}$

$= \frac{1}{2\pi \times 25 \times 22.91} = 277.8\ \mu F$

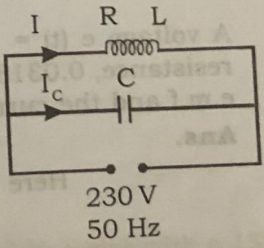
Current $I = \frac{250}{10} = 25\ A$

72. An inductor of 0.5H inductance 90 resistance is connected in parallel with a 20μF capacity. A voltage of 230V at 50Hz is maintained across the circuit. Determine the total power taken from the source.

Ans. $R = 90\ \Omega$ $L = 0.5\ H$ $C = 20 \times 10^{-6}\ F$

$X_L = 2\pi \times 50 \times 0.5 = 157.07\ \Omega$

$X_C = \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}} = 159.15\ \Omega$



Impedance of coil $= 90 + j\,157.07\ \Omega$

Impedance of capacitor $= 0 - j159.15\ \Omega$

Impedance of parallel combination $= \frac{Z_1 \times Z_2}{Z_1 + Z_2}$

$Z = 320.01\angle -28.49\ \Omega$

$\cos\phi = \frac{R}{Z} = \frac{281.25}{320} = 0.878$ leading

Total current $I = \frac{230}{320.01} = 0.718\ A$

Power $= 230 \times 0.718 \times 0.878 = 145.14$ watts

73. An alternating voltage 80+j60 is applied to a circuit and the current flwing is -4+j 10 A. Find (i) the impedance of the circuit (ii) the power consumed

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and (iii) the phase angle.

Ans. $V = 80 + j60\ V = 100\angle 36.86\ V$

$I = -4 + j10\ A = 10.77\angle 111.8\ V$

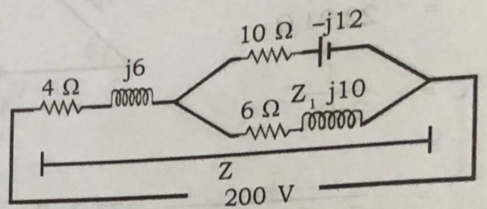
(i) $Z = \frac{100\angle 36.86}{10.77\angle 11.8} = 9.284\angle -74.93\ \Omega$ or $2.41 - j8.96\ \Omega$

Power factor $= \frac{R}{Z} = \frac{2.41}{9.284} = 0.259$

(ii) Power $= 100 \times 10.77 \times 0.259 = 279.5$ Watts

(iii) Phase angle between voltage and current $= \cos^{-1} 0.259 = 74.98^0$

74. For the circuit in Fig. find current I.



Ans. Impedance of the parallel combination $Z = \frac{(10 - j12)(6 + j10)}{(10 - j12) + (6 + j10)}$

$$= \frac{(15.62\angle -50.19) \times (11.66\angle 59.03)}{(10 - j12) + (6 + j10)} = \frac{182.16\angle 8.84}{(16 - j2)} = \frac{180 + j28}{16 - j2}$$

$$= 10.861 + j3.107\ \Omega$$

Total impedance $Z = (4 + j6) + (10.861 + j3.107) = 14.86 + j9.107\ \Omega$

$$= 17.43\angle 31.5\ \Omega$$

Total current $I = \frac{200\angle 0}{17.43\angle 31.5} = 11.47\angle -31.5\ A$

75. A voltage $e(t) = 10 \sin 314\ t$ is applied to a series circuit consisting of 10 resistance, 0.0318 H inductance and a capacitor of 63.6 μF. Find the frequency e.m.f and the current in the circuit.

Ans. $e(t) = 10 \sin 314\ t$

Here $\omega = 314 = 2\pi f$

$f = \frac{314}{2\pi} = 49.97\ Hz \approx 50\ Hz$

$V_{rms} = \frac{10}{\sqrt{2}} = 7.07\ V$

$X_L = 2\pi f L = 2\pi \times 50 \times 0.0318 = 9.99\ \Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 63.6 \times 10^{-6}} = 0.04\ \Omega$

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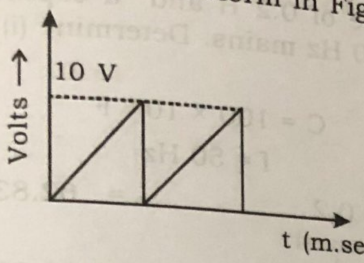
Impedance $Z = \sqrt{10^2 + (9.99 - 50.04)^2} = 41.27\ \Omega$

Current $I = \frac{7.07}{41.27} = 0.171\ A$

76. For the periodic voltage waveform in Fig. determine.



(i) frequency of the waveform.
(ii) Wave equation for 0<t< 100 m.sec.
(iii) r.m.s. value
(iv) Average value.
(v) Form factor

Ans. (i) Frequency 'f' $= \frac{1}{T} = 10\ Hz$

(ii) Wave equation e $= \frac{10}{100 \times 10^{-3}} \times t$

(iii) Mean square value $e^2 = \frac{1}{100 \times 10^{-3}} \int_0^{100\times10^{-3}} \frac{10^2}{(100 \times 10^{-3})^2} \times t^2 dt$

$$= \frac{100}{(100 \times 10^{-3})^3} \left(\frac{t^3}{3}\right)_0^{100\times10^{-3}} = 33.3\ V$$

$$V_{rms} = \sqrt{33.3} = 5.77\ V$$

(iv) Average value $= \frac{V_M + 0}{2} = \frac{10}{2} = 5V$

(v) Forms factor $= \frac{V_{rms}}{V_{av}} = \frac{5.77}{5} = 1.15$

77. A coil of resistance 10 Ω and inductance 0.1 H is connected in series with a 150 μF capacitor acros a 200V, 50 Hz supply. Calculate (i) the inductive reactance; (ii) the capacity reactance, (ii) the impedance, (iv) the current, and (v) the power factor.

Ans. (i) $X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.4\ \Omega$

(ii) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.22\ \Omega$

(iii) $Z = R + j\,(X_L - X_C) = 10 + j\,(31.41 - 21.22) = 14.27\angle 45.54\Omega$

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(iv) $I = \frac{200}{14.27} = 14.01$ A

(v) $\cos\phi = \frac{R}{Z} = \frac{10}{14.27} = 0.7$ lagging

78. A resistance of 20 Ω, an inductance of 0.2 H and a capacitance of 100μF are connected in series across 220 V, 50 Hz mains. Determine (i) the impedance and (ii) the current.

Ans. R = 20 Ω L = 0.2 H C = 100 × 10^{-6} F

Supply voltage = 220 V f = 50 Hz

$X_L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

$X_C = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\ \Omega$

Impedance $Z = \sqrt{20^2 + (63.83 - 31.830)^2} = 36.89\ \Omega$

Current $I = \frac{220}{36.89} = 5.96$ A

79. A sinusoidal voltage V=50 sinωt is applied to a series RL circuit. The current in the circuit is given by i=25 sin (ωt – 53⁰). Determine (a) apparent power; (b) power factor ; (c) average power.

Ans. v = 50 sin ωt, i = 25 sin (ωt – 53)

In this R –L circuit current lags voltage by an angle 53^0.

So power factor = cos 53^0 = 0.601 lagging

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35$ V

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{25}{\sqrt{2}} = 17.67$ A

Apparent power = 35.35 × 17.67 = 624.9 A

Average power = 35.35 × 17.67 × 0.601 = 375.4 W

80. Find the form factor and peak factor of a current waveform i=10 sin (100 πt+ 30^0) + 15 cos (100 πt – 30^0) A.

Ans. 15 cos (100 πt – 30) = 15 sin (100 πt + 60)

i = 10 sin (100 πt + 30) + 15 sin (100 πt + 60)

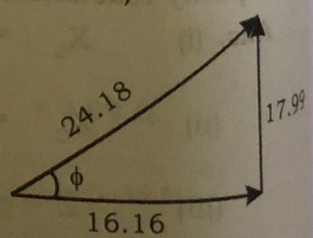
= 10 (sin 100 πt × cos 30) + 10 cos 100 πt × sin 30) + 15 (sin 100 πt × cos 60) + 15 (cos 100 πt × sin 60)

= 16.16 sin 100 πt + 17.99 cos 100 πt

$\sqrt{16.16^2 + 17.99^2} = 24.18$

Dividing both sides by using 24.18

$\frac{i}{24.18} = \frac{16.16}{24.18}\sin 100\pi t + \frac{17.99}{24.18}\cos 100\pi t$



From the figure $\frac{16.16}{24.18} = \cos\phi$ and $\frac{17.99}{24.18} = \sin\phi$

$i = 24.18\,[\cos\phi \sin 100\,\pi t + \sin\phi \cos 100\,\pi t] = 24.18 \sin(100\,\pi t + \phi)$

$\phi = \cos^{-1}\left(\frac{16.16}{17.99}\right) = 48.06^0$

Then $i = 24.182 \sin(100\,\pi t + 48.06)$

$$I_m = 24.182$$

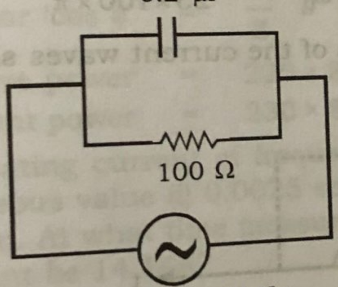
$$I_{rms} = \frac{24.182}{\sqrt{2}} = 17.09$$

$$I_{av} = \frac{2 \times 24.182}{\pi} = 15.39$$

$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = \frac{17.09}{15.39} = 1.11$$

$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{24.182}{17.09} = 1.414$$

81. Determine the total current, phase angle and total impedance for the circuit shown in Fig..



Ans.

$$X_C = \frac{1}{2\pi \times 5 \times 10^3 \times 0.2 \times 10^{-6}} = 159.1\ \Omega$$

$$I_C = \frac{20\angle 0}{159.1\angle -90} = 0.125\angle 90\ \text{A}$$

$$I_R = \frac{20}{100} = 0.2\ \text{A}$$

$$I = I_C + I_R = 0.235\angle 32^0\ \text{A}$$

Phase angle = Angle between voltage and current $= 32^0$

Total impedance $Z = \frac{20\angle 0}{0.235\angle 32} = 84.8\angle -32$

Or $= 71.91 - j44.94\ \Omega$

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82. A series R-L-C circuit is connected across a V= 150 sin 100 πt volts. The maximum current in the circuit is 25A and at this condition voltage across the capacitor is 600 V. Find the numerical values of circuit elements.

Ans. Under resonance conditions, $I_m = \frac{V}{R}$ and $V_L = V_C$.

$$V_{rms} = \frac{150}{\sqrt{2}}$$

$$R = \frac{V_{rms}}{I_m} = \frac{150}{\sqrt{2} \times 25} = 4.24\ \Omega$$

$$V_C = V_L = 600\ V$$

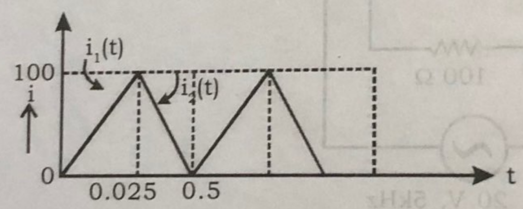
$$V_C = I_m \times X_C = \frac{I_m}{\omega_0 C}$$

$$C = \frac{I_m}{\omega_0 C} = \frac{25}{100 \times \pi \times 600} = 0.13\ mF$$

$$V_L = I_m \times X_L = I_m \times \omega_0 L$$

$$L = \frac{V_L}{I_m \times \omega_0} = \frac{600}{25 \times 100 \times \pi} = 76.4\ mH$$

83. Find the RMS and average values of the current waves shown in Fig.



i_1 (t) = 4000 t, 0<t< 0.025
i_2 (t) = 100 e^{-1000}(t-0.025), 0.025 <t<0.05.

Ans. Rms value of current (I_{rms}) $= \sqrt{\int_0^{0.025} \frac{i^2 \cdot dt}{0.025}}$

Substituting the value of i $= \frac{I_m t}{0.025} = \frac{100\,t}{0.025}$

$$I_{rms} = \sqrt{\int_0^{0.025} \frac{\left(\frac{100\,t}{0.025}\right)^2 \cdot dt}{0.025}} = \frac{100}{\sqrt{3}} = 57.7\ A$$

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Average value I_{av} $= \int_0^{0.025} \frac{i \cdot dt}{0.025}$

Substituting the value of i $= \frac{I_m \cdot t}{0.025}$

$I_{av} = \int_0^{0.025} \frac{100 \cdot t}{0.025} dt = \frac{100}{2} = 50$ A

84. Two impedance $25.23\angle 37^0$ and $18.65\angle -68^0$ are connected in series across 230 V, 50 Hz supply. Find the current, power factor, active power and apparent power.

Ans. $Z_1 = 25.23\angle 37^0\ \Omega = 20.15 + j15.18\ \Omega$

$Z_2 = 18.65\angle -68^0\ \Omega = 6.98 - j17.29\ \Omega$

Total impedance $Z = Z_1 + Z_2 = 27.21\angle -4.44\ \Omega = 27.13 - j2.1\ \Omega$

Current $I = \frac{230\angle 0}{27.21\angle -4.44} = 8.45\angle 4.45$ A

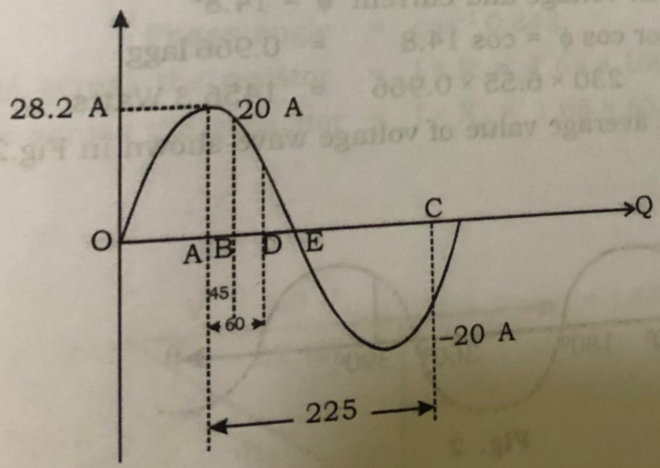
Power factor 'cos ϕ' $= \frac{R}{Z} = \frac{27.13}{27.21} = 0.997$ leading

Active power $= 230 \times 8.45 \times 0.997 = 1937.6$ W

Apparent power $= 230 \times 8.45 = 1943.5$ VA

85. A sinusoidal alternating current of frequency 50Hz has an r.m.s value of 20 A. Find the instantaneous value (i) 0.0025 sec. (ii) 0.0125 sec after passing through a positive maximum. At what time measured form the positive maximum, will the instantanous current be 14.14A?

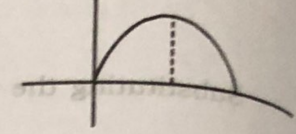
Ans.



$I_{rms} = 20$ a, $f = 50$ Hz, $I_m = \sqrt{2} \times I_{rms} = \sqrt{2} \times 20 = 28.28$

$i = I_m \sin \omega t = 28.28 \sin \omega t$

Since the time values are given from the point where the current has positive maximum value, the equation becomes,



$$i = I_m \cos\omega t$$

(i) When $t = 0.0025$ second

$$i = 28.2 \cos 18000 \times 0.0025 = 20\text{A}$$

(ii) At $t = 0.0125$ second

$$i = 28.2 \cos 18000 \times 0.0125 = -20\text{ A}$$

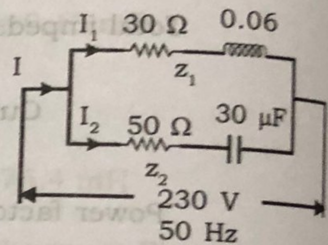
(iii) When $i = 14.14$ A

$$14.14 = 28.2 \cos (18000t)$$

$$t = \frac{1}{300} \text{ second}$$

86. A two branch parallel circuit consists of R=30 Ω L = 0.06 H in series in branch I and R =50 Ω, C 30μF in series in branch 2. The circuit is connected to 230V, 50 Hz supply. Find (i) current in each branch, (ii) total current and (iii) power and power factor.

Ans. $X_L = 2\pi \times 50 \times 0.06 = 18.8\ \Omega$



$$X_C = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.103\ \Omega$$

$$Z_1 = 30 + j\,18.84\ \Omega$$

$$Z_2 = 50 - j106.10\ \Omega$$

(i) Then $I_1 = \dfrac{230\angle 0}{30 + j18.84} = 6.49\angle -32.12\text{ A}$

$$I_2 = \frac{230\angle 0}{50 - j106.1} = 1.96\angle 64.76\text{A}$$

(ii) Total current $I = I_1 + I_2 = 6.55\angle -14.8\text{A}$

Phase angle between voltage and current 'ϕ' = 14.8°

So power factor $\cos\phi = \cos 14.8 = 0.966$ lagg

$$\text{Power} = 230 \times 6.55 \times 0.966 = 1456.3 \text{ Watts}$$

87. Find the r.m.s and average value of voltage wave shown in Fig.2.

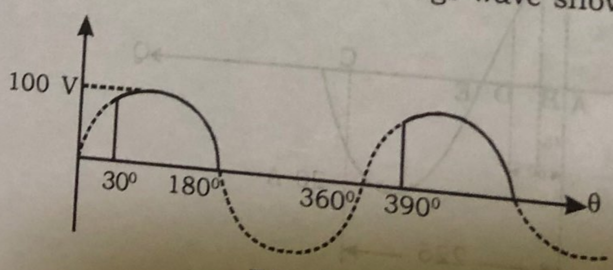


Fig. 2

Ans. Mean square value = Area under squared wave ÷ Base

$$= \int_{\pi/6}^{\pi} \frac{100^2 \sin^2\theta}{2\pi} d\theta = \frac{100^2}{2\pi} \int_{\pi/6}^{\pi} \sin^2\theta \, d\theta$$

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$$= \frac{100^2}{2\pi}\int_{\pi/6}^{\pi}\frac{1-\cos 2\theta}{2}\,d\theta = \frac{100^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/6}^{\pi} = \frac{100^2}{4\pi}\times 3.051$$

$$\text{Rms value} = \sqrt{\frac{100^2}{4\pi}\times 3.051} = 49.2\text{ V}$$

$$E_{av} = \frac{1}{2\pi}\int_{\pi/6}^{\pi}100\sin\theta\,d\theta = \frac{100}{2\pi}[-\cos\theta]_{\pi/6}^{\pi} = \frac{100}{2\pi}[-\cos 180 + \cos 30]$$

$$= \frac{100}{2\pi}\left[1+\frac{\sqrt{3}}{2}\right] = 29.69\text{ V}$$

88. A resistance of 100 Ω in series with 50 μF (micro farad) capacitor is connected to supply of 200V, 50 Hz. Find (i) impedance, current, power factor and phase angle and (ii) voltage across the resistance and capacitor. Draw phasor diagram.

Ans. $R = 100\ \Omega,\ C = 50 \times 10^{-6}$ F

Frequency = 50 Hz

Voltage applied = 200 V

$$X_C = \frac{1}{2\pi\times 50\times 50\times 10^{-6}} = 63.66\ \Omega$$

(i) Impedance $Z = 100 - j63.66 = 118.54\angle -32.48$

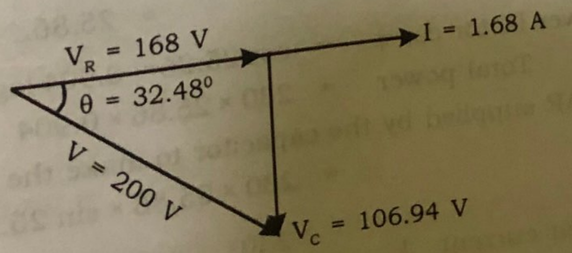
$$I = \frac{200}{118.54} = 1.68\text{ A}$$

Power factor 'cos ϕ' $= \frac{R}{Z} = \frac{100}{118.54} = 0.8435$ leading

Phase angle $= \cos^{-1} 0.843 = 32.48^0$

(ii) Voltage across the resistor $= I \times R = 1.68 \times 100 = 168$ V

Voltage across the capacitor $= I \times X_C = 1.68 \times 63.66 = 106.94$ V



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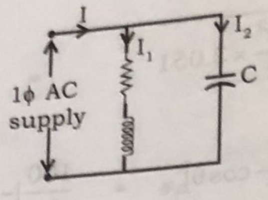
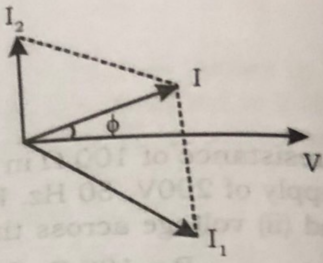
89. An R-L circuit is connected in parallel with a capacitor C. This combination is supplied from a sinusoidal voltage source. Find an expression for the current drawn from the supply. Draw phasor diagrams.

Ans.



$$Z_1 = R + jX_L, \quad Z_2 = -jX_C$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}; \quad I = \frac{V}{Z}$$



Using current divider rule

$$I_1 = \frac{Z_2 I}{Z_1 + Z_2} \quad \text{and} \quad I_2 = \frac{Z_1 I}{Z_1 + Z_2}$$

90. Two impedance $14 + j5\ \Omega$ and $18 + j\ 12\ \Omega$ are connected in parallel across a 230 V, 50 Hz supply. Determine (i) the admittance of each branch and of the entire circuit (ii) total current, power, power factor (iii) the capacitance which when connected in parallel with the above circuit make the overall power factor unity.

Ans. $Z_1 = 14 + j5\ \Omega = 14.86\angle 19.65\ \Omega$

$Z_2 = 18 + j12\ \Omega = 21.63\angle 33.7\ \Omega$

Admittance $Y_1 = \frac{1}{Z_1} = \frac{1}{14.86\angle 19.65}$

$= 0.0633 - j0.0226$ Siemens

Admittance $Y_2 = \frac{1}{Z_2} = \frac{1}{21.63\angle 33.7}$

$= 0.0384 - j0.0256$ Siemens

Total admittance $Y = Y_1 + Y_2 = 0.1017 - j0.048$ Siemens

Current 'I' $= V \times Y = 230[0.1017 - j0.048]$

$= 25.86\angle -25.26$ A

Power factor $\cos\phi = \cos 25.26 = 0.904$ lagging

Total power $= 230 \times 25.86 \times 0.904 = 5376.8$ Watts

(ii) Reactive kVAR supplied by the capacitor to make the power factor unity

$= 230 \times 25.86 \times \sin 25.26^0 = 2.538$ kVAR

Capacitor current $I_C = \frac{230}{X_C} = 230 \times 2\pi \times 50 \times C$

$2.538 \times 10^3 = 230 \times (230 \times 2\pi \times 50 \times C)$

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Then, capacitance required 'C' = 152.7 μF

91. Find the real and reactive power supplied from a 230 V,50Hz supply to a load of 10 Ω resistance in series with 100 μF capacitance. What is the power factor?

Ans. R = 10 Ω, C = 100 × 10^{-6} F, f = 50 Hz, V = 230 V

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times\pi\times 50\times 100\times 10^{-16}} = 31.83\ \Omega$$

$$\text{Total impedance } Z = \sqrt{10^2+(31.83)^2} = 33.36\ \Omega$$

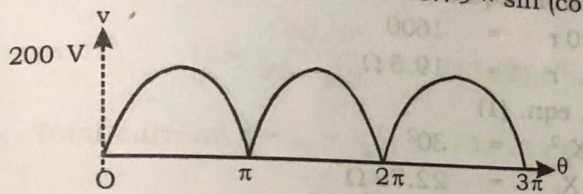
$$\text{Current } I = \frac{230}{33.36} = 6.79\text{ A}$$

$$\text{Power factor 'cos }\phi\text{'} = \frac{R}{Z} = \frac{10}{33.36} = 0.299 \text{ leading}$$

$$\text{Real power} = 230 \times 6.79 \times 0.299 = 466.9\text{ W}$$

$$\text{Reactive power} = 230 \times 6.79 \times \sin(\cos^{-1} 0.299) = 1489.8\text{ VAR}$$

92.



Find the average value, r.m.s value and form factor of the periodic waveform shown in Fig.

Ans. E_m = 200 V

Equation of the wave form = 200 sin θ

(i) E_{av} = Area under the wave over one half cycle ÷ Base

$$= \int_0^{\pi} \frac{200\sin\theta}{\pi} d\theta = \frac{200}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{200}{\pi}[-\cos\theta]_0^{\pi} = \frac{200}{\pi}(2) = 127.38\text{ V}$$

$$\text{(ii) } E_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} 200^2 \sin^2\theta\, d\theta} = \sqrt{\frac{200^2}{\pi}\int_0^{\pi}\sin^2\theta\, d\theta} = \sqrt{\frac{200^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{200^2}{\pi}\left(\frac{\theta-\sin 2\theta}{2}\right)_0^{\pi}} = \sqrt{\frac{200^2}{\pi}(\pi-0)} = \sqrt{\frac{200^2}{2}} = 141.42$$

$$\text{(iii) Form factor} = \frac{E_{rms}}{E_{av}} = \frac{141.42}{127.38} = 1.111$$

93. A current of 5 A flows through a non-inductiven resistance in series with a coil when supplied at 250 V, 50 Hz. If the voltage across the resistance is 125 V and across the coil is 150 V, calculate:

(i) The impedance, reactance and resistance of the coil,

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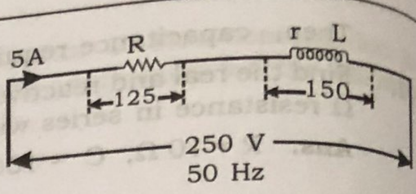


(ii) Power absorbed by the coil, and
(iii) Total power.
Draw phasor diagram.

Ans. V = 250 V, I = 5A, V_R = 125 V

$$R = \frac{125}{5} = 25\ \Omega$$

$$V_L = 150\ V$$

Impedance of the coil $Z_1 = \frac{150}{5} = 30\ \Omega$

Total impedance $Z = \frac{250}{5} = 50\ \Omega$

$r^2 + X_L^2 = 30^2$ ....(1) Equation for coil impedance

$(25 + r)^2 + X_L^2 = 50^2$ ....(2) Equation for total impedance

Substracting equation (1) from equation (2)

We get, $25^2 + 50\ r = 1600$

$r = 19.5\ \Omega$

Put the value of 'r' on eqn. (1)

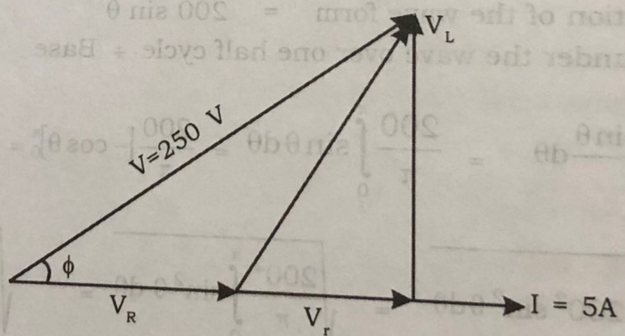
$19.5^2 + X_L^2 = 30^2$

$X_L = 22.79\ \Omega$

(ii) Power consumed by coil $= I^2 \times r = 5^2 \times 19.5 = 487.5$ W

(iii) Total power $= I^2 \times (r + R) = 5^2 \times (25 + 19.5)$
$= 1112.5$ Watts

Phasor diagram



94. A coil having a resistance of 45 Ω and an inductance of 0.4H is connected in parallel with a capacitor having a cpacitance of 20 micro-farad across a 230 V, 50 Hz supply. Calculate (a) Current taken from the supply, (b) Power factor of the combination, and (c) The total energy absorbed in 3 hours.

Ans. R = 45 Ω, L = 0.4 H, C = 20 × 10^{-6} F, V = 230 V, f = 50 Hz

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.4 = 125.66\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}} = 159.15\ \Omega$$

Impedance of the coil 'Z_1' $= 45 + j125.66\ \Omega$

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Impedance of capacitor $Z_2 = 0 - j159.15\ \Omega$

Total impedance $Z = \dfrac{Z_1 \times Z_2}{Z_1 + Z_2} = 378.4\angle 17.01\ \Omega$

(a) $I = \dfrac{230}{378.4} = 0.607$ A

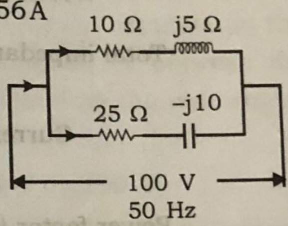
(b) Power factor 'cos ϕ' = cos 17.01 = 0.956 lagging

(c) Energy absorbed for 3 hors = $230 \times 0.607 \times 0.956 \times 3 = 400$ Wh

= 0.4 KWh

95. A parallel circuit with 2 branches has R = 10 Ω X_L = 5 Ω in series in branch 1 and R = 25 Ω X_C = 10 Ω in series in branch 2. The circuit is fed by 100 V, 50 Hz source. Find (i) current in each branch (ii) total impedance and current (iii) power and power factor.

Ans. (i) Current in branch (1) $I_1 = \dfrac{100\angle 0}{10 + j5} = 8.94\angle -26.56$ A



$$I_2 = \frac{100\angle 0}{25 - j10} = 3.713\angle 21.8^0 \text{ A}$$

(ii) Total current, $I = I_1 + I_2 = 11.73\angle -12.83^0$ A

Total impedance, $Z = \dfrac{V}{I} = \dfrac{100\angle 0}{11.73\angle -12.88^0} = 8.51\angle 12.88\ \Omega$

(iii) Power factor cos ϕ $= \dfrac{R}{Z} = \dfrac{8.3}{8.51} = 0.97$ lagging

Power = $100 \times 11.73 \times 0.97 = 1137.81$ Watts

96. An alternating wave is given by $i = 100\,e^{-200\,t}$. The time period of the wave is 0.05 sec. Find the average and r.m.s. values of the wave.

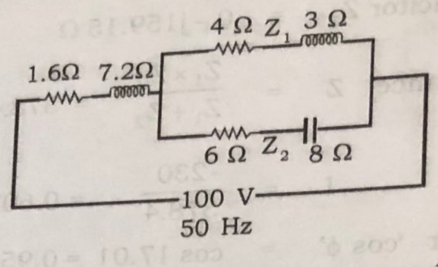
Ans. Average value $= \dfrac{1}{0.05}\displaystyle\int_0^{0.05} 100\,e^{-200t}dt = \dfrac{100}{0.05(-200)}[e^{-200\,t}]_0^{0.05} = 10$ A

$$[\text{RMS value}]^2 = \frac{1}{0.05}\int_0^{0.05} (100e^{-200t})^2.dt = \frac{10000}{0.05}\int_0^{0.05} e^{-400t}.dt$$

$$= \frac{10000}{-400 \times 0.05}[e^{-400\,t}]_0^{0.05} = 500$$

RMS value $= \sqrt{500} = 22.36$ A

97. For the circuit shown in Fig. given below. Find the total impedance, current, power and power factor.



Ans. Impedance of parallel combination Z_1 and $Z_2 = \dfrac{Z_1 \times Z_2}{Z_1 + Z_2}$

$$Z_1 = 4 + j3\ \Omega = 5\angle 36.86^0\ \Omega$$

$$Z_2 = 6 - j8\ \Omega = 10\angle -53.13^0\ \Omega = \frac{(5\angle 36.86) \times (10\angle -53.13)}{(4 + j3) + (6 - j8)} = 4.4 + j0.8\ \Omega$$

$$= 4.47\angle 10.3\Omega$$

Total impedance $Z = (1.6 + j7.2) + (4.4 + j0.8) = 6 + j8\ \Omega = 10\angle 53.13^0$ A

$$\text{Current, } I = \frac{100\angle 0}{10\angle -53.13^0} = 10\angle -53.13^0 \text{ A}$$

$$\text{Power factor } (\cos\phi) = \frac{R}{Z} = \frac{6}{10} = 0.6 \text{ lagging}$$

$$\text{Power} = 100 \times 10 \times 0.6 = 600 \text{ Watts}$$

98. In an a.c. circuit consisting of two elements in series, the equations of voltage and current are given by e = 50 sin (200t – 25) and i = 8 sin (2000t + 5). Calculate the frequency, power factor and the values of circuit constants.

Ans. e = 50 sin (2000t – 25), i = 8 sin (200t + 5), $\omega = 2\pi f = 2000$

$$\text{Then } f = \frac{2000}{2\pi} = 318.3 \text{ Hz.}$$

Phase angle between voltage and current = 25 + 5 = 30^0

So, power factor = cos 30 = 0.866 leading

$$E_m = 50,\ I_m = 8,\ E_{rms} = \frac{50}{\sqrt{2}} = 35.35,\ I_{rms} = \frac{8}{\sqrt{2}} = 5.65$$

$$Z = \frac{E_{rms}}{I_{rms}} = \frac{35.35}{5.65} = 6.25\ \Omega = 6.25\angle -30^0 = 5.41 - j3.12$$

From this, R = 5.41 Ω and $X_C = 3.12$

$$C = \frac{1}{2\pi f X_C} = 0.16 \text{ mF}$$

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PRACTICE PROBLEMS

1. An alternating voltage is triangular in shape, rising at a constant rate to a maximum of 300 V in 8 ms and then falling to zero at a constant rate in 4 ms. The negative half cycle is identical in shape to the positive half cycle. Calculate (a) the mean voltage over half a cycle and (b) the r.m.s voltage. [*Ans.*(a) 150 V, (b) 170 V]

2. An alternating voltage is represented by $v = 20 \sin 157.1\, t$ volts. Find (a) the maximum value (b) the frequency (c) the periodic time (d) what is the angular velocity of the phasor representing the waveform.

   [*Ans.* (a) 20 V (b) 25 Hz (c) 0.04s (d) 157.1 rad /s]

3. A coil takes a current of 5A from a 20 V d.c. supply. When connected to a 200 V, 50 Hz a.c. supply the current is 25 A. Calculate (a) the resistance (b) impedance and (c) inductance of the coil. [*Ans.* (a) 4 Ω (b) 8 Ω (c) 22.05 mH]

4. A capacitor C is connected in series with a 40Ω resistor across a supply of frequency 60 Hz. A current of 3A flows and the circuit impedance is 50 Ω. Calculate (a) the value of capacitance, C (b) the supply voltage (c) the phase angle between the supply voltage and current (d) the p.d across the resistor and (e) the p.d. across the capacitor.

   [*Ans.* (a) 88.42 μF (b) 150 V (c) 36.87° leading (d) 120 V (e) 90 V]

5. A 50 μF capacitor is connected to 100 V, 200 Hz supply. Determine the true power and the apparent power. [*Ans.* 0, 628.3 VA]

6. A 200 V, 60 Hz supply is applied to a capacitive circuit. The current flowing is 2A and the power dissipated is 150 W. Calculate the values of the resistance and capacitance. [37.5 Ω, 28.61 μF]

7. A coil of resistance 400 Ω and inductance 0.2 H is connected to a 75 V, 400 Hz supply. Calculate the power dissipated in the coil. [*Ans.* 5.452 W]

8. A capacitor C is connected in parallel with a resistance R across 60 V, 100 Hz supply. The supply current is 0.6 A at a power factor of 0.8 leading. Calculate the values of R and C. [*Ans.* R =125 Ω, C = 9.55 μF]

9. In a particular circuit, a voltage of 10 V at 25 Hz produces a current of 100 mA, while the same voltage at 75 Hz produces 60 mA. Draw the circuit diagram and insert the values of the components. [*Ans.* 88.277Ω, 0.3 H]

10. Two impedances Z_1 and Z_2 are connected in series across 200 V, 50 Hz a.c supply. The total current drawn by the series combination is 2.3 A. The p.f of Z_1 is 0.8 lagging. The voltage drop across Z_1 is twice the voltage across Z_2 and it is in 90° out of phase with it. Determine (a) the value of Z_2 (b) power consumed by Z_2 and (c) total power consumed by the circuit.

    [*Ans.* (a) 38.38 Ω (b) 123.43 W (c) 452.578 W]

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